Local Subsystems on the Light Front: Luminosity and Local Amplitudes

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Outline

First part: Non-perturbative quantisation of gravitational subsystem at local null hypersurfaces (inside spacetime).

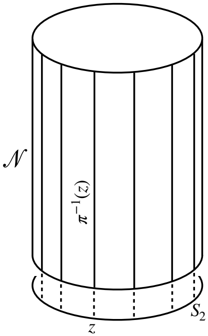
Second part: How loop quantum discreteness of geometry can create a bound on radiated power.

Outlook: A framework (top-down, theory independent) for building local amplitudes.

Gravitational Subsystems on the Null Front: The Case of the γ-Palatini–Holst Action

Spacetime region bounded by null surface:

- Compact spacetime region $\mathscr{M}$.
- Bounded by spacelike disks M_0, M_1 and null surface $\mathscr{N}$.
- Null surface boundary $\mathscr{N}$ embedded into abstract bundle (ruled surface) $P(\pi, \mathscr{C}) \simeq \mathbb{R} \times \mathscr{C}$.
- Null generators $\pi^{-1}(z)$.



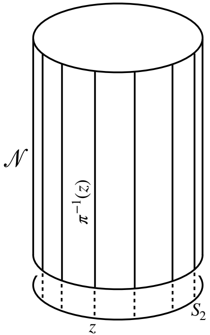
Signature $(0{+}{+})$ metric.

$$q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \qquad i,j = 1,2.$$

Parametrisation of the dyad

$$\boxed{e^i = \Omega\, S^i{}_m\, e^m_{(o)}.}$$

- Conformal factor Ω parametrizes the overall scale.
- $SL(2,\mathbb{R})$-Holonomy $S^i{}_m$ determines the shape degrees of freedom.
- Fiducial background dyad $e^j_{(o)}$.



We consider a null strip $\mathscr{N}$ with two corners as our subsystem. No unique clock along $\mathscr{N}$. Convenient choice

Boundary condition at $\partial\mathscr{N} = \mathscr{C}_+ \cup \mathscr{C}_-$,

$$\mathscr{U}(\partial\mathscr{N}, z, \bar{z}) = \pm 1,$$

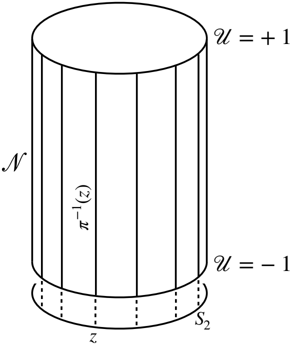
Affinity proportional to expansion

$$\partial^b_{\mathscr{U}} \nabla_b \partial^a_{\mathscr{U}} = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\Omega^2)\partial^a_{\mathscr{U}}$$

Parametrize physical clock $\mathscr{U}$ relative to unphysical coordinate u.

$$\partial_u \mathscr{U} = \mathrm{e}^{\chi}$$

The *chronoton* χ becomes a quantum reference frame (part of phase space).



Diffeomorphisms with compact support that deform the null generators are gauge redundancies.

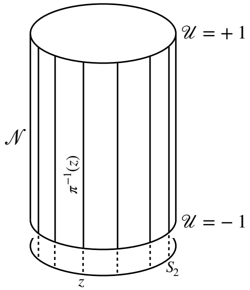
- Remove them by only considering fibre preserving diffeos.
- Residual diffeomorphisms: angle dependent reparametrizations of $\mathscr{U}$.
- Canonical generator on phase space: Raychaudhuri equation.

Raychaudhuri equation

$$\frac{\mathrm{d}^2}{\mathrm{d}\mathscr{U}^2}\Omega^2 = -2\,\sigma\bar{\sigma}\,\Omega^2 \mathrm{e}^{-2\chi}.$$

$SL(2,\mathbb{R})$ holonomy

$$\frac{\mathrm{d}}{\mathrm{d}u}S = \Big(\varphi J + \big(\sigma\bar{X} + \mathrm{cc.}\big)\Big)S.$$



$SL(2,\mathbb{R})$ generators split into $U(1)$ complex structure J and shear generators:

$$[J,X] = -2\mathrm{i}\,X, \qquad [J,\bar{X}] = +2\mathrm{i}\,\bar{X}, \qquad [X,\bar{X}] = \mathrm{i}\,J.$$

Phase space and action

In $D = 4$, there are two Lorentz scalars that we can build from the curvature tensor:

$$R[A, e] = F^{\alpha\beta}{}_{ab}[A] e_\alpha{}^a e_b{}^\beta,$$

$$R^*[A, e] = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta ab}[A] e_\mu{}^a e_\nu{}^b \approx 0.$$

Therefore, in the first-order formalism, there are *two coupling constants* at linear order in the curvature,

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4v \left[R - \frac{1}{\gamma} R^*\right] + \text{boundary terms.}$$

G is Newton's constant, γ is the Barbero–Immirzi parameter.

How does γ affect the quantisation of charges and radiation?

Starting from the γ-action for GR, we obtain null symplectic structure

$$\begin{aligned}\Theta_{\mathscr{N}} = \frac{1}{2\mathrm{i}}\int_{\partial\mathscr{N}} d^2 v_o\,(a\mathbb{d}\bar{a} - \mathrm{cc.}) + \\ + \int_{\mathscr{N}} d^3 v_o\, p_\chi \mathbb{d}\chi + \frac{1}{2\mathrm{i}}\int_{\mathscr{N}} d^3 v_o\,\big(b\mathbb{d}\bar{b} - \mathrm{cc.}\big) + \\ + \int_{\mathscr{N}} d^3 v_o\,\mathrm{Tr}_{\mathfrak{sl}(2,\mathbb{R})}\big(\Pi\mathbb{d}SS^{-1}\big)\,.\end{aligned}$$

Geometric interpretation:

- Area operator becomes number operator: $\Omega^2\big|_{\mathscr{C}_\pm} = 8\pi\gamma G\, a\bar{a}$.
- Expansion turns into number operator: $\frac{\mathrm{d}}{\mathrm{d}u}\Omega^2 = 8\pi\gamma G\, b\bar{b}$.
- Chronoton modes: $p_\chi = \frac{1}{8\pi G}\,\frac{\mathrm{d}}{\mathrm{d}u}\Omega^2$ (a constraint).
- Last line: $\mathfrak{su}(1,1)$ shape modes.
- GR phase space: these variables plus constraints (all polynomial).

Among the canonical variables are *shape modes* $S \in SL(2,\mathbb{R}) \simeq SU(1,1)$.

- The conjugate momentum is $\Pi^A{}_B \in \mathfrak{su}(1,1)$.
- Utilize (fermionic) bosonic representation

$$\Pi_{AB} = \pi_{(A}\omega_{B)} \equiv \frac{1}{2}\left(\pi_A\omega_B + \pi_B\omega_A\right).$$

- Fundamental Poisson brackets

$$\begin{aligned}\left\{\pi_A(u,\zeta,\bar{\zeta}), \omega_B(u',\zeta,\bar{\zeta})\right\} &= +\epsilon_{AB}\delta_{\mathcal{N}}(u-u',\zeta-\zeta',\bar{\zeta}-\bar{\zeta}'),\\ \left\{\underset{\sim}{\pi}_A(u,\zeta,\bar{\zeta}), \underset{\sim}{\omega}_B(u',\zeta,\bar{\zeta})\right\} &= -\epsilon_{AB}\delta_{\mathcal{N}}(u-u',\zeta-\zeta',\bar{\zeta}-\bar{\zeta}').\end{aligned}$$

- Reconstruction of $S \in SU(1,1)$

$$\underset{\sim}{\omega}^A = [S^{-1}]^A{}_B\omega^B, \qquad \underset{\sim}{\pi}^A = [S^{-1}]^A{}_B\pi^B.$$

Hamilton constraint

There are first-class and second-class constraints. Among the first-class constraints is the Hamiltonian constraint, which is

$$H[N] = \frac{\mathrm{i}}{2}\int_{\mathcal{N}} d^3 v_o\, N(\bar{b}\dot{b} - \mathrm{cc.}) + \int_{\mathcal{N}} d^3 v_o\, p_\chi(N\dot{\chi} + \dot{N}) + \\ + \frac{1}{2}\int_{\mathcal{N}} d^3 v_o\, N\left(\pi_A \dot{\omega}^A - \dot{\pi}_A \omega^A - \underset{\sim}{\pi}_A \dot{\underset{\sim}{\omega}}^A + \dot{\underset{\sim}{\pi}}_A \underset{\sim}{\omega}^A\right).$$

Basic idea:

1. The Hamiltonian $H[N]$ generates a Virasoro algebra.
   See also recent results by Freidel and Ciambelli.
2. Idea: Utilize CFT methods to quantize this algebra.
   Mode expansion, positive (negative) frequency modes, Fock vacuum etc.
3. Problem: What selects the statistics of the oscillators?
4. Hint: The abelian current $j = {:}\pi_A \omega^A{:}$ is the square root of the $SU(1,1)$ Casimir.

Ashtekar–Lewandowski vacuum

Introduce tessellation of null surface cuts (thickend null rays).

- Smearing $(p_\chi, \chi, b, \pi_A, \omega^A \dots)$

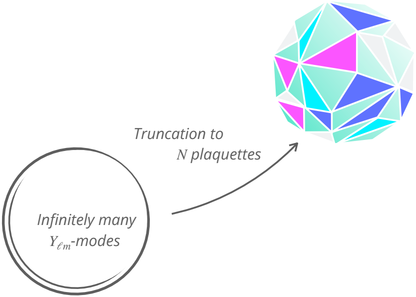
$$p_\chi(i) = \int_{\mathscr{C}_i} d^2 v_o \, p_\chi,$$

$$\chi(i) = \chi(x_i), \quad x_i \in \mathscr{C}_i, \quad \textit{etc.}$$

- Mode expansion

$$\chi(i) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \chi_n(i) \mathrm{e}^{-inu},$$

$$p_\chi(i) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} p_{\chi n}(i) \mathrm{e}^{-inu}.$$



Ashtekar–Lewandowski (no geometry) vacuum for all modes, e.g.

$$\boxed{\chi_n(i)|0\rangle = p_{\chi n}(i)|0\rangle = \dots = 0, \quad n > 0, i = 1, \dots, N.}$$

$SU(1,1)$ Casimir

Canonical momentum dual to the shape modes:

$$\Pi = LJ + c\bar{X} + \bar{c}X \in \mathfrak{su}(1,1)$$

$SU(1,1)$ Casimir in terms of the geometric data:

$$L^2 - c\bar{c} = \frac{1}{(16\pi\gamma G)^2}\Omega^4\left(\vartheta^2 - 4(1+\gamma^2)\sigma\bar{\sigma}\right).$$

What we find is:

- Bose statistics for $(\pi_A, \omega^A, b, \bar{b})$:
  - CFT has negative central charge.
  - Both $L^2 \leq c\bar{c}$ and $L^2 \geq c\bar{c}$ possible.
  - But resulting CFT is non-unitary.
- Fermi statistics for $(\pi_A, \omega^A, b, \bar{b})$:
  - CFT has positive central charge.
  - Only $L^2 \geq c\bar{c}$ *infra-Planckian* modes occur.
  - violation of unitarity can be avoided.

Luminosity bound

On physical grounds (unitarity), we are led to choose Fermi statistics.

It is easy to check that this implies the inequality

$$\vartheta^2 - 4(1+\gamma^2)\sigma\bar{\sigma} \geq 0.$$

For semi-classical states, we should thus get (as expectation values)

$$\frac{\sigma\bar{\sigma}}{\vartheta^2} \leq \frac{1}{4}\frac{1}{1+\gamma^2}.$$

This must hold for all null hypersurfaces.

Caveat: We do not have constructed such semi-classical states explicitly.
In here, we merely assume they exist.

Utilize Bondi expansion to characerize gravitational radiation.

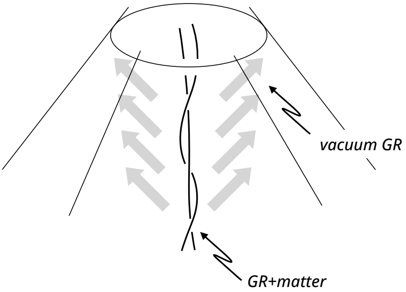
- Bondi mass loss formula

$$\dot{M}_{\mathrm{B}}(u) = -\frac{1}{4\pi G}\oint_{S^2_u \subset \mathscr{I}_+} d^2 v_o \, \dot{\sigma}^{(0)} \dot{\bar{\sigma}}^{(0)}.$$

- Falloff conditions

$$\sigma_{(\ell)}(u,r,\zeta,\bar{\zeta}) = -\frac{\dot{\sigma}^{(0)}(u,\zeta,\bar{\zeta})}{r} + \mathcal{O}(r^{-2}),$$

$$\vartheta_{(\ell)}(u,r,\zeta,\bar{\zeta}) = -\frac{2}{r} + \mathcal{O}(r^{-2}).$$



Asymptotic luminosity from quasi-local observables

$$\mathscr{L}_{\mathrm{B}}(u,\zeta,\bar{\zeta}) = \frac{4c^5}{G}\lim_{r\to\infty}\frac{\bar{\sigma}_{(\ell)}(u,r,\zeta,\bar{\zeta})\sigma_{(\ell)}(u,r,\zeta,\bar{\zeta})}{(\vartheta_{(\ell)}(u,r,\zeta,\bar{\zeta}))^2} \leq \frac{c^5}{G}\frac{1}{1+\gamma^2}.$$

In the S-matrix approach, the $\mathcal{O}(r^{-1})$ term in $\vartheta_{(\ell)}$ is a commuting c-number. In the quasi-local quantisation of gravity, it becomes a q-number akin to LQG area operator.

Planck scale luminosity

Humanity has come close to observing such power

$$\mathscr{L}_{\mathrm{P}} = \frac{c^5}{G} \approx 3,63 \times 10^{52}\,\mathrm{W},$$

$$\mathscr{L}_{peak}\Big|_{\mathrm{GW150914}} \approx 3,6 \times 10^{49}\,\mathrm{W}.$$

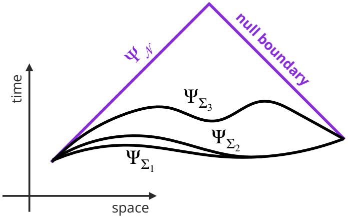
Side remark: Only in $D = 4$ spacetime dimensions, the Planck power (luminosity) is independent of $\hbar$

$$\mathscr{L}_{\mathrm{P}} = \frac{m_{\mathrm{P}} c^2}{t_{\mathrm{P}}} = \frac{\hbar^{\frac{D-4}{D-2}} c^{\frac{2D+2}{D-2}}}{G^{\frac{2}{D-2}}}.$$

Top-Down Approach:
Local Amplitudes from Local Subsystems

How to get rid of the Wheeler–De Witt (WDW) equation

- Initial data: three-metric h_{ab} and extrinsic curvature $\tilde{\pi}^{ab} \sim K_{ab} \sim \dot{h}_{ab}$.
- Constraints $\mathscr{H}[h, \tilde{\pi}] = 0$ and $\mathscr{H}_a[h, \tilde{\pi}] = 0$ generate gauge redundancies on phase space.
- Gauge redundancies: states on Σ_1, Σ_2, $\ldots$ are gauge equivalent.



- Basic idea: Characterize the entire gauge equivalence class $[\Psi_{\Sigma_i}]$ by pushing the time-evolution (gauge) to its extreme.
- The boundary of the future Cauchy development of Σ_i is a null boundary. We saw how the problem simplifies. Less constraints. Perhaps even physical consequences (luminosity).

[Ashtekar, Speziale, Reisenberger, Freidel, Donnelly, Ciambelli, Leigh, Geiller, Pranzetti, ..., Donney, Grumiller, Fiorucci, Ruzziconi, ..., Riello, Hoehn, Carrozza, ... , Barnich, Prabhu, Chandrasekaran, Flanagan, Compère, ...]

Local Amplitudes from Quantum Characteristic Gluing

Promise of the Wheeler–De Witt equation: no time evolution, all dynamics to be extracted from physical states.

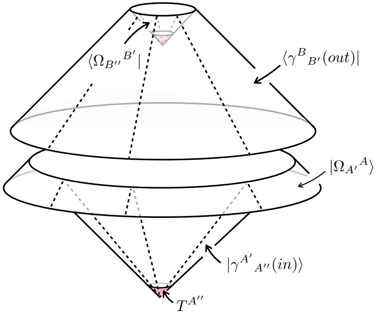
- One incarnation: Covariant LQG, spinfoams

$$W[\Psi_{\partial\mathscr{M}}] = \langle\psi_{out}|\boldsymbol{P}|\psi_{in}\rangle.$$

- Kinematical states on a null slab

$$|\gamma^{A}{}_{A'}\rangle \in \mathscr{K}_{edge} \otimes \mathscr{K}_{bulk} \otimes \mathscr{K}^{*}_{edge}.$$

- Assuming existence of *tip states* T^A, vacuum states $|\Omega^A{}_{A'}\rangle$ and the projector $\boldsymbol{P}$, we can formally introduce local amplitudes.



- Basic idea: Add thin strips to upper and lower cones. Data on the *in* and *out* cones are diffeomorphic. Upon projecting kinematical onto physical states, we obtain proposal for amplitudes.

$$W(\gamma(in)\to\gamma(out)) = \langle\Omega,\uparrow| \otimes \langle\gamma(out),\downarrow|\boldsymbol{P}|\gamma(in),\uparrow| \otimes |\Omega,\downarrow\rangle.$$

Conclusion

Take home messages

- Non-perturbative quantisation of null initial data at finite distance.
  - Spectra for geometric observables reproduce LQG discretenss of area.
  - Difference of the area at initial and final cut: number operator.
  - Turning on γ, we activate otherwise irrelevant $SU(1,1)$ representations.
- Local amplitudes from gluing local subsystems on the light front.
- We strengthened earlier conjecture on Planck luminosity bound.
- Results implicitly proof that $r \to \infty$ and $\hbar \leftarrow 0$ may not commute.