On the Cauchy Horizon (In)Stability of Regular Black Holes

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Antonio Panassiti
in collaboration with Alfio Bonanno and Frank Saueressig

Università di Catania and Radboud University

Content

Motivations

- (Simplified) big picture:
  the ones we see in nature are black holes or regular black holes?

1)Previous analyses in the literature focus only on a portion of the phase space of the dynamical system corresponding to the Ori model. Importance of the initial conditions for the perturbation.

Motivations

2)In general, why studying the CH instability? Can we cure it?

- a)It seems that regular black holes commonly imply the presence of the CH
- b)It is a crucial theoretical open problem and an open problem of internal "consistency"
- c)It is related to the destiny of the cosmic censorship conjecture
- d)It is related to geodesic completeness in a (regular) black hole spacetime
- e)It can tell us something about the astrophysical viability of (regular) black holes

Interrelated problems

CH instability in a nutshell

- The CH is a surface of infinite blueshift

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2\ ,\ \ \{t, r, \theta, \varphi\}$$ (Units: $c = 1,\ G_N = 1$)

with $f(r) \equiv 1 - 2M(r)/r = 0$ having two different roots, $r_{\rm EH}$ and $r_{\rm CH}$.

$\downarrow$

$p_\mu = (E, p_r, 0, 0)$ for γ,

$u_\mu = (\tilde{E}, u_r, 0, 0)$ for ,

$$E_{obs.}(r) = p_\mu u^\mu = f(r)^{-1}\left[E\tilde{E} - E\sqrt{\tilde{E}^2 - f(r)}\right]$$

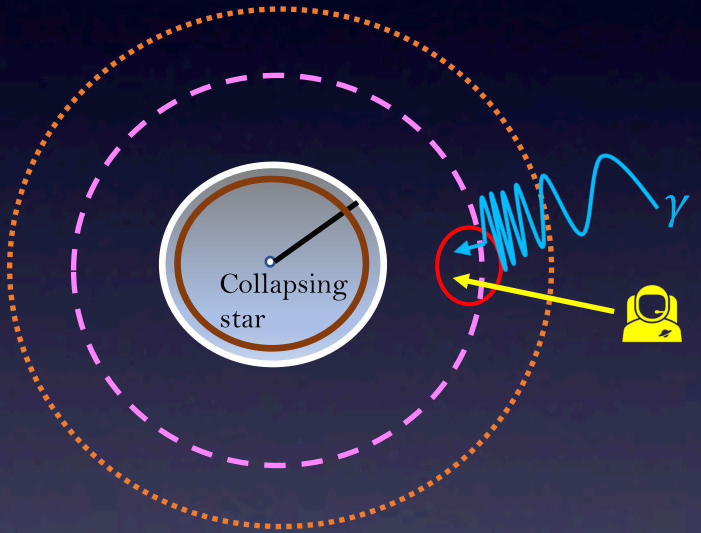
$\Downarrow$

Then, at the meeting point:

$$\lim_{r \to r_{\rm CH}} E_{\rm obs.}(r) \sim \lim_{f(r)\to 0^-} \frac{2E\tilde{E}}{f(r)} = +\infty$$

$\Downarrow$

The system composed by (regular) black hole + incoming photon faces an ultraviolet catastrophe.



Event horizon

Cauchy horizon

Incoming photon

Free falling observer

CH instability and mass inflation

Incoming flux: incoming perturbation. ⊕ Outgoing flux: portion of the originally incoming perturbation backscattered by the black hole's curvature near the CH. $\Longrightarrow$ intersection of two fluxes.

General Relativity and Reissner-Nordström $f(r) = \left(1 - \frac{2m_0}{r} + \frac{e^2}{r^2}\right)$

with event horizon r_+ and CH r_- and surface gravity at r_-: $\kappa_- \equiv -\frac{1}{2}\frac{\partial f(r)}{\partial r}\Big|_{r_-} > 0$

$\downarrow$

Advaced Eddington-Finkelstein coordinates $\{v, r, \theta, \varphi\}$, with $v = t + r^*$:

$ds^2 = -f_\pm(r, v_\pm)dv_\pm^2 + 2drdv_\pm + r^2 d\Omega^2$ where $f_\pm = 1 - 2M_\pm(r, v_\pm)/r$

$\downarrow$

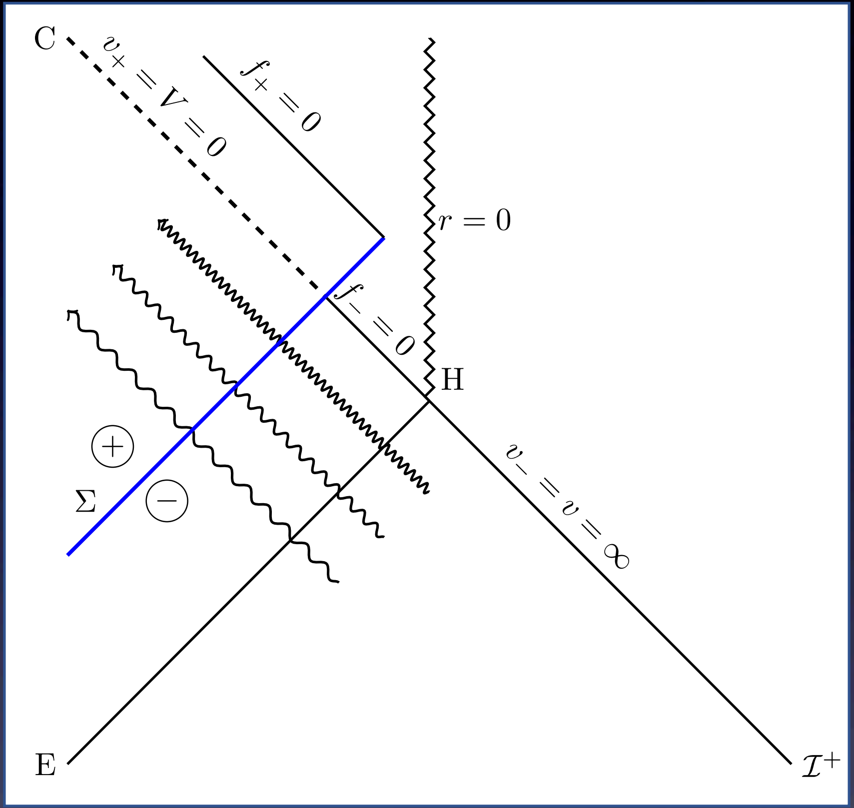
Boundary condition at the event horizon: $m_-(v) = m_0 - \frac{\beta}{v^{p-1}}$, $\beta > 0$, $p \geq 12$ (Price's law)

$\Downarrow$

At the CH: $m_+(v) \backsimeq c\, v^{-p} e^{\kappa_- v}$ $M_+(v) \propto v^{-p} e^{\kappa_- v}$ $K_+(v) \propto v^{-2p} e^{2\kappa_- v}$

$\Longrightarrow$ *A curvature singularity builds up at the CH*

Ori model



$G_{\mu\nu} = T_{\mu\nu}^{eff}$ and a background geometry $f(r)$

↓

Incoming perturbation: continuos flux of radiation

Outgoing perturbation: spherical shell Σ of radiation

↓

1° Dynamical equation: $\dot{R}(v) = \frac{1}{2} f_- \Big|_\Sigma$ for $R(v)$

2° Dynamical equation: $\left(\frac{1}{f_+}\frac{\partial M_+}{\partial v}\right)\Big|_\Sigma = F(v)$ for $m_+(v)$

Notation:

$F(v) \equiv \left(\frac{1}{f_-}\frac{\partial M_-}{\partial v}\right)\Big|_\Sigma$, $R(v) \equiv \textit{shell position}$, $\dot{y} \equiv dy/dv$

$M(v) \equiv \textit{perturbed mass function}$

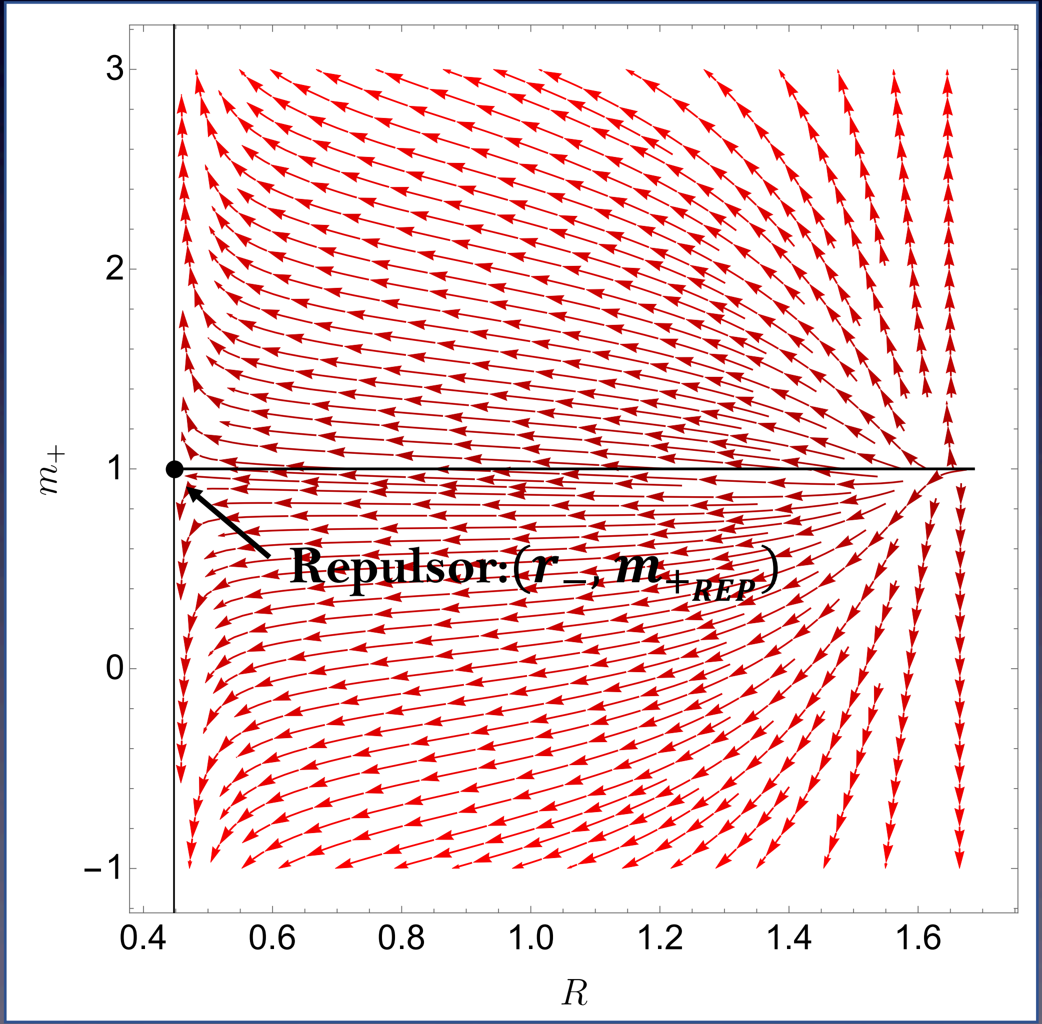
↓

- Free parameters: m_0, β, p
- Degrees of freedom: $R(v),\ m_+(v)$
- Independent variable: v

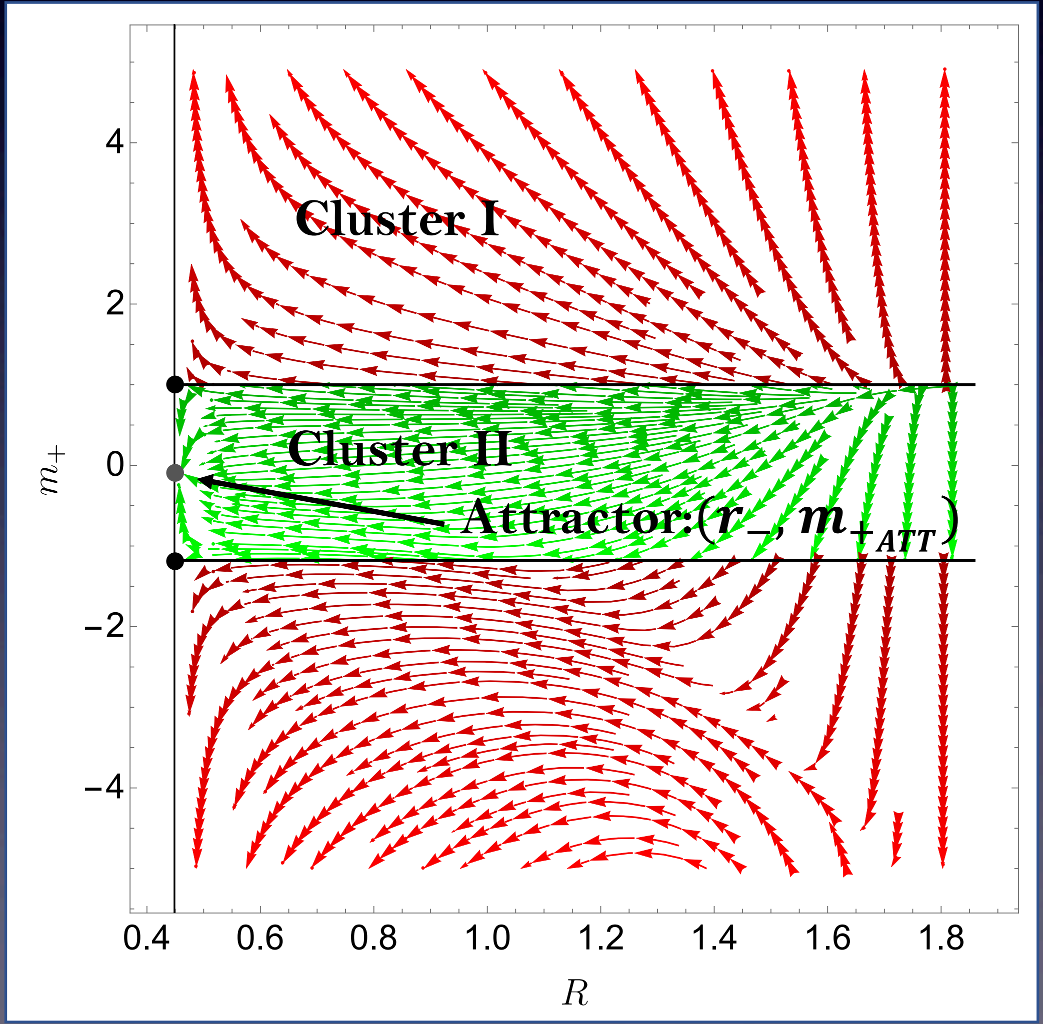
Assigning a spacetime $M(r) \leftrightarrow f(r)$,
and initial conditions $R(v_i),\ m_+(v_i)$,
the system and its evoluton are fully determined

Phase space for the CH (in)stability of regular black holes

Bardeen solution (also Reissner-Nordström)



Solution from asymptotically safe collapse



Phase space for the CH (in)stability of regular black holes

Step 1

quasi - Fixed points of the dynamical system:

1° eq. $\dot{R}(v) = 0$

2° eq. $\dot{m}(v) = 0$

Step 2

Analytical solutions around the fixed points, using Frobenius ansatz

Step 3

Numerical integrations of the full equations

Repulsor and mass inflation

Bardeen $f(r) = \left[1 - \frac{2m_0 r^2}{(r^2 + a^2)^{3/2}}\right]$

$\kappa_- = 1$

$m_0 = 1$

$a = 0.586$

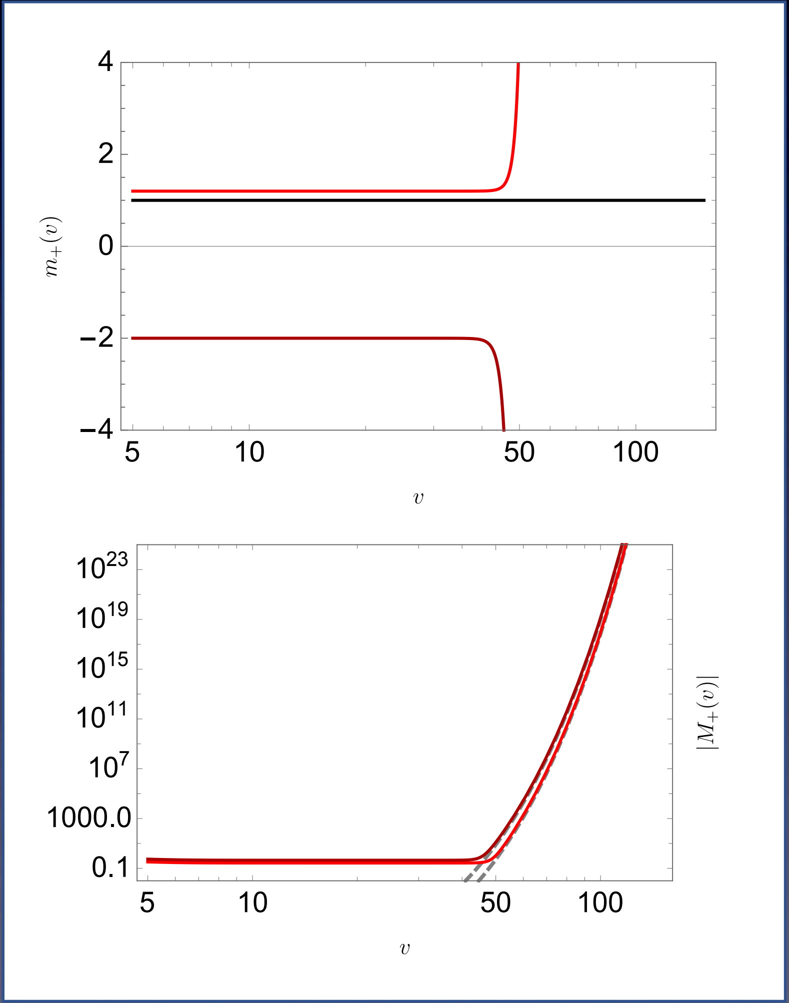
$\beta = 1$

$p = 12$

quasi - Fixed Points:

1° eq. $\dot{R}(v) = 0$

2° eq. $\dot{m}(v) = 0$



Ending State

$m_+(v) \simeq \pm c\, v^{-p} e^{\kappa_- v}$

$M_+(v) \simeq \pm c \frac{r_-^3}{(r_-^2 + a^2)^{3/2}} v^{-p} e^{\kappa_- v}$

$K_+(v) \propto v^{-2p} e^{2\kappa_- v}$

Attractor and mass inflation avoidance

AS-collapse $f(r) = \left[1 - \frac{r^2}{12\xi} log\left(1 + \frac{6\xi m_0}{r^3}\right)^2\right]$

$\kappa_- = 1$

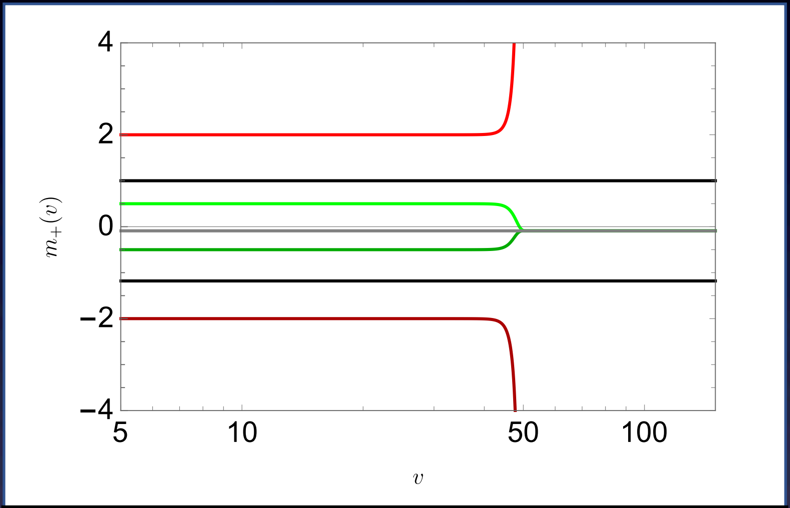
$m_0 = 1$

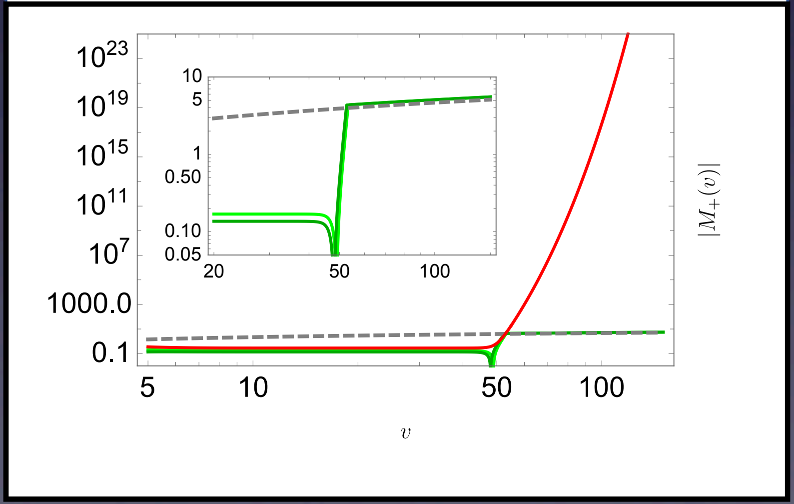
$\xi = 0.167$

$\beta = 1$

$p = 12$

$M_+(v) \sim \mathcal{O}(m_0)$ at all v





Cluster II Ending State

$m_+(v) \backsimeq \frac{r_-^3}{6\xi}[x(v) - 1]$

$x(v) \equiv \frac{y_0[1 + \sum_{n=1} y_n log^n(v)]}{v^p}$

Cluster II

$M_+(v) \backsimeq \frac{r_-^3}{12\xi} log(y_0 v^{-p})^2$

Cluster II

$K_+(v) \propto v^{4p}$

Singularity strength: traversability of the CH

$\mathcal{T}(\tau) \equiv \int^{\tau} d\tau' \int^{\tau''} d\tau'' |C_+^2(\tau'')| \longrightarrow$ "Tipler weak" singularity if $\lim_{\tau \longrightarrow 0} \mathcal{T}(\tau) < \infty \longrightarrow$ tidal forces do not diverge

$\mathcal{K}r(\tau) \equiv \int^{\tau} d\tau' |C_+^2(\tau')| \longrightarrow$ "Królak weak" singularity if already $\lim_{\tau \longrightarrow 0} \mathcal{K}r(\tau) < \infty \longrightarrow$ expansion does not diverges negatively

	Bardeen solution	AS-collapse Cluster I	AS-collapse Cluster II
$M_+(v)$	$\propto \pm v^{-p} e^{\kappa_- v}$	$\propto v^{-p} e^{\kappa_- v}$	$\propto -\log v^p$
$K_+(v)$	$\propto v^{-2p} e^{2\kappa_- v}$	$\propto v^{-2p} e^{2\kappa_- v}$	$\propto v^{4p}$
Stable	✗	✗	✓
$C_+^2(v)$	$\propto v^{-2p} e^{2\kappa_- v}$	$\propto v^0$	$\propto v^{4p}$
Traversable	$\mathcal{T}(\tau)$ ✓	✓	$\mathcal{K}r(\tau)$ ✓

It should be pointed out that any classical extension beyond the mass-inflation singularity will require an infinite ingoing flux of negative energy along the CH. This

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"Inner Structure of a Charged Black hole: an Exact Mass-Inflation Solution", A. Ori, PRL (1991)

Conclusion and outlook

Results

- Mass inflation instability is related to the presence, in the phase space, of a repulsive fixed point
- The presence of the instability strongly depends on the specific functional form of the BH geometry
- For regular BH, an attractor in the phase space can provide a resolutive mechanism

However

- Application of the Ori model to regular BHs imply non-trivial assumptions
- In our solution with stable mass function, the Kretschmann scalar is still unstable

Conclusion

- Mass inflation at the CH seems not universal

Possibilities and Outlook

- Quenched divergences, stability, weak singularities, traversability, geodesic completeness at the CH *may* be possible. Then perturbed regular BH *may* actually remain regular.

- This calls for proposals for the type of *physics beyond the CH* itself

References

New Regular BH from AS-collapse

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"Dust collapse in asymptotic safety:
a path to regular black holes",
A. Bonanno, D. Malafarina, A. P. ,
PRL 132 (2024) 3, 031401

⇓

More elaboration on this idea

"non-Singular Spacetime Cores via Gravitational Evanescence of Collapsing Matter",
A. P. , to appear soon on arXiv

Cauchy Horizon and Regular BHs

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"Cauchy horizon (In)Stability
of Regular Black Holes",
A. Bonanno, A. P. , F. Saueressig,
arXiv:2507.03581

Thank you for your attention!

and free Palestine

Modified action for our non-singular collapse model

Theory:

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\,[R + 2\chi(\epsilon)\mathcal{L}]$$

A scalar function of the energy-density, which couples matter and geometry non-minimally, with property: $\chi(\epsilon = 0) = 8\pi G_N$

Matter Lagrangian

Matter source:

$$T_{\mu\nu} = [\epsilon + p(\epsilon)]u_\mu u_\nu + p g_{\mu\nu}$$

Field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\partial(\chi\epsilon)}{\partial\epsilon} T_{\mu\nu} + \frac{\partial\chi}{\partial\epsilon}\epsilon^2 g_{\mu\nu}$$

$$\equiv T^{eff}_{\mu\nu}$$

$$8\pi G(\epsilon) \equiv \frac{\partial(\chi\epsilon)}{\partial\epsilon} \qquad \Lambda(\epsilon) \equiv -\frac{\partial\chi}{\partial\epsilon}\epsilon^2$$

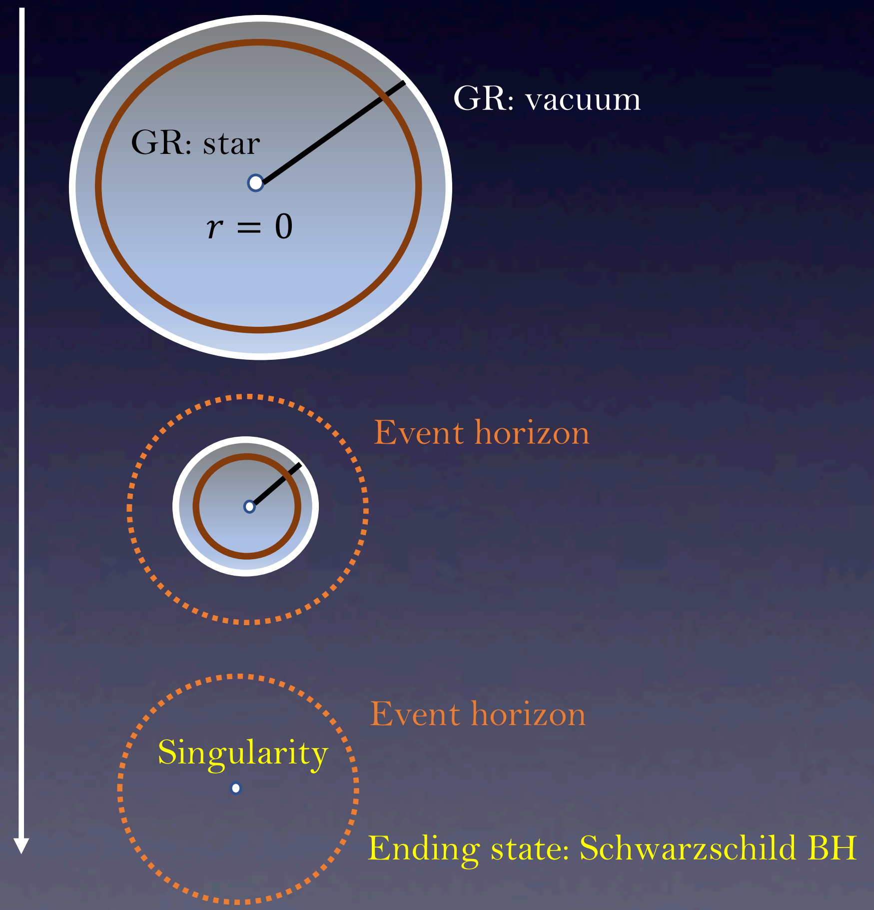
Choice of $G(\epsilon)$:

$$G(k) = \frac{G_N}{1 + \frac{G_N}{g_*} k^2}$$

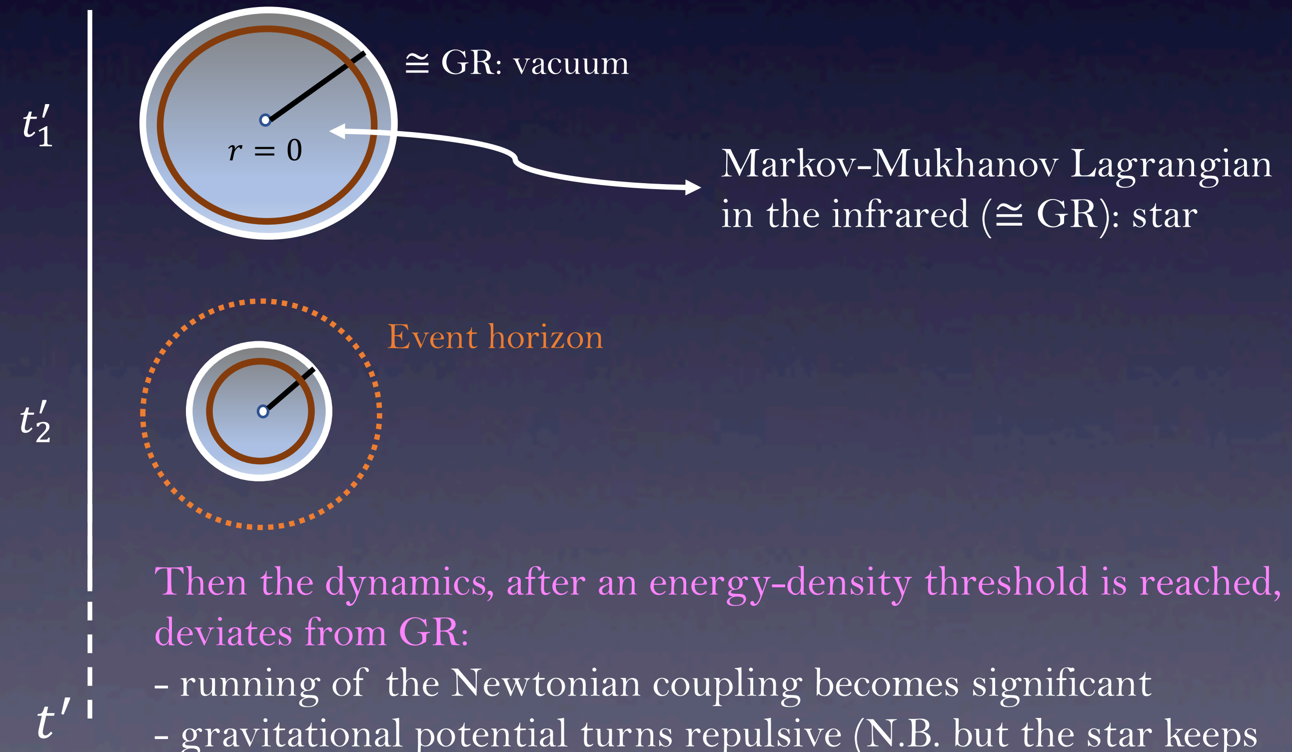
$$G(\epsilon) = \frac{G_N}{1+\xi\epsilon}$$

Model of asymptotically safe gravitational collapse

Oppenheimer-Snyder collapse in General Relativity:
gravitational collapse → Schwarzschild BH



Our model of collapse implementing the idea of an *asymptotically safe gravitational interaction* (by means of a modified classical theory of gravity):
gravitational collapse → A new *regular* BH



Then the dynamics, after an energy-density threshold is reached, deviates from GR:

- running of the Newtonian coupling becomes significant
- gravitational potential turns repulsive (N.B. but the star keeps contracting)
- a hypothesis of the singularity theorem is violated

$$\mathcal{M} = \mathcal{M}_{star} \cup \mathcal{M}_{exterior}$$

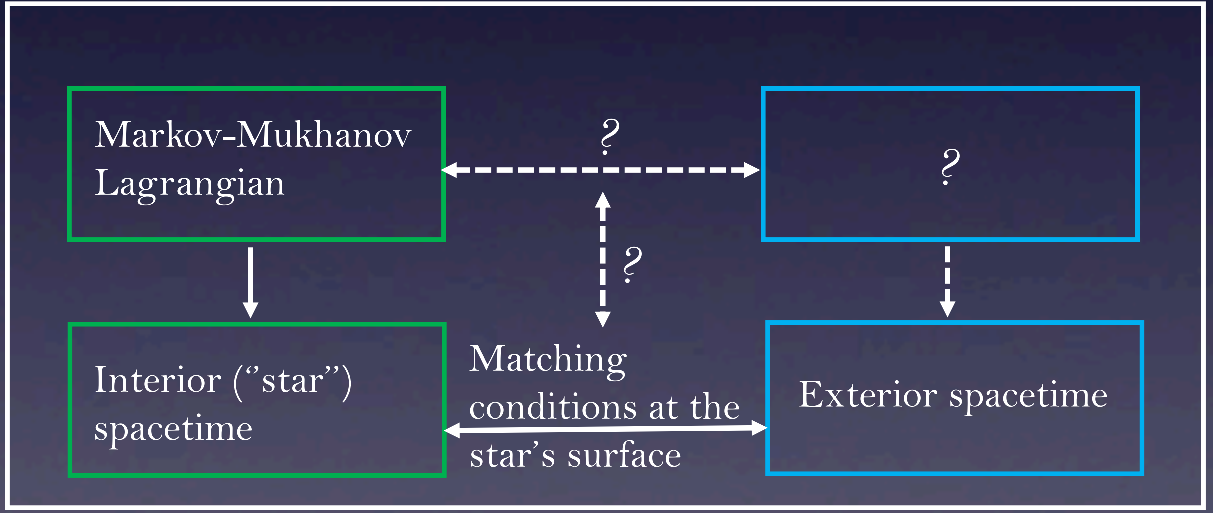
$$ds^2_{star} = -dt^2 + a(t)^2 dr^2 + a(t)^2 r^2 d\Omega^2 \qquad \{t, r, \theta, \varphi\} \qquad 0 \le r \le r_b$$

$$\frac{da}{dt} = -\sqrt{\frac{\log(1+3m_0\xi/a^3)}{3\xi}}\, a^2 \qquad a(t) \sim e^{-\frac{t^2}{4\xi}}, \quad t \to \infty$$

$$ds^2_{exterior} = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2 d\Omega^2 \qquad \{T, R, \theta, \varphi\}$$

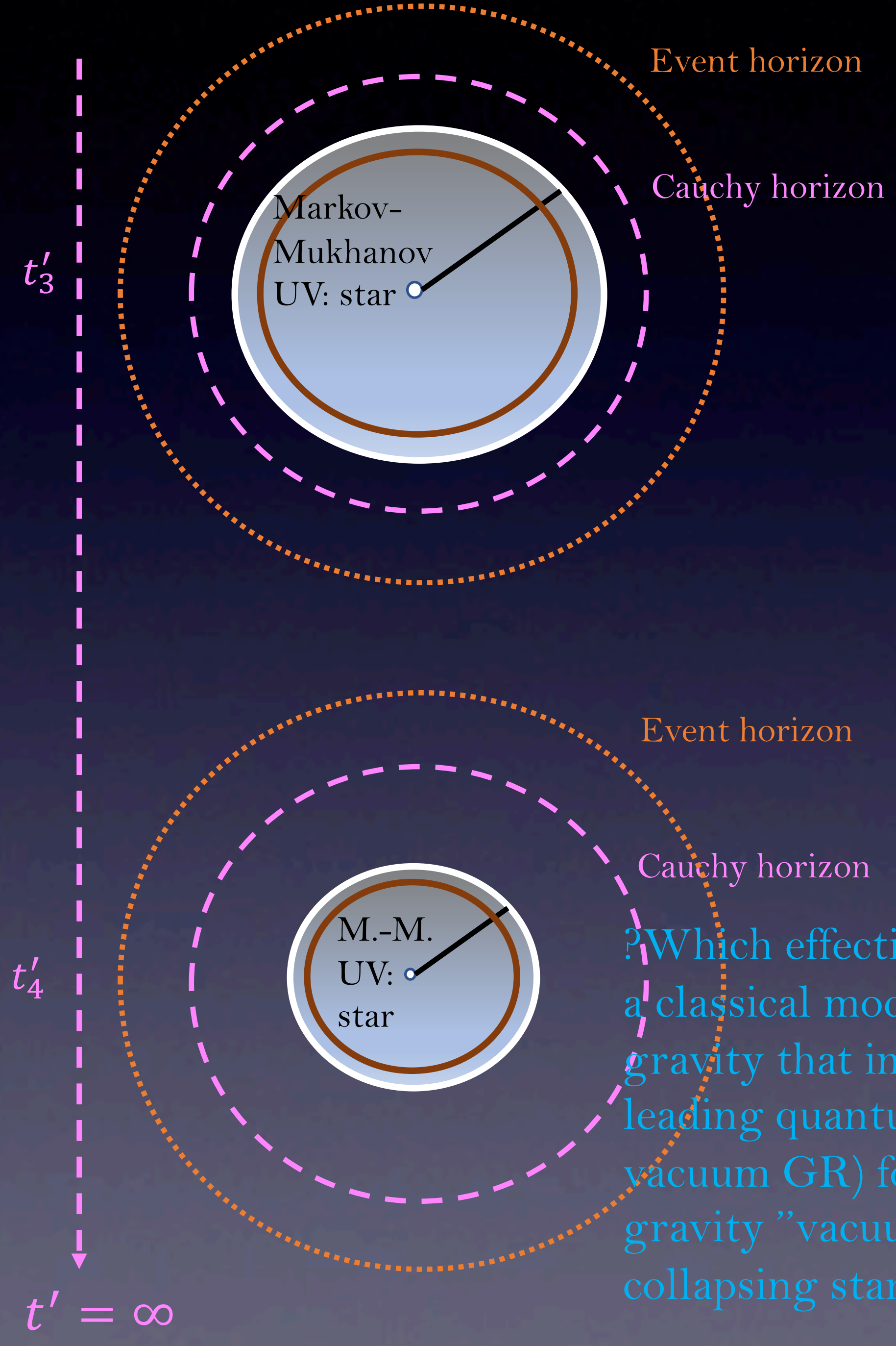
$$R_b \le R < +\infty$$

$$f(R) \equiv \left[1 - \frac{2R^2}{6\xi} \log\left(1 + \frac{6M_0}{R^3}\xi\right)\right] \qquad R \ge R_b(T) = r_b a(t) > 0$$



Ending state is that there is no ending state: an ongoing "*eternal collapse*" in the core

(while there would be a bounce for positive intrinsic curvature)



?Which effective theory (i.e. a classical modified theory of gravity that implements the leading quantum correction to vacuum GR) for the strong-gravity "vacuum" (i.e. outside the collapsing star)?