Celestial holography from Twistor space

Lionel Mason

The Mathematical Institute, Oxford, `lmason@maths.ox.ac.uk`

Jurek memorial meeting, Warsaw, 16/9/2025

On: 2407.04028, 2506.01888 with Adam Kmec, Romain Ruzziconi, Atul Sharma & Akshay Yelleshpur-Srikant. $\leftrightarrow$ Friedel, Pranzetti, Raclariu,... & older work with: Adamo & Sharma 2103.16984, 2110.06066, 2203.02238, Dunajski, 00s.

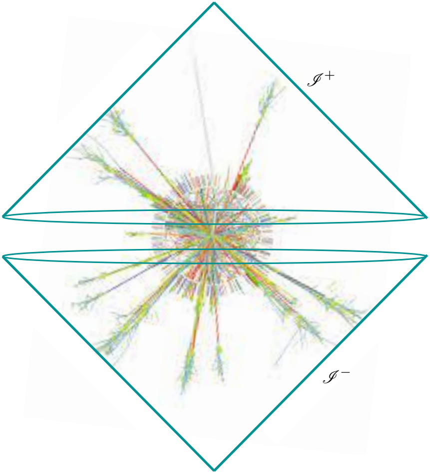
Celestial Holography

[Strominger, Pasterski, Puhm, Raclariu, Kapec, Guevara, Himwich, Pate, Costello, Paquette, ...]

Seek holographic dual to quantum gravity theories in asymptotically flat space-times.

The S-matrix is most natural observable.

- Specify initial & final states of massless particles at $\mathscr{I}^{\pm} = S^2 \times \mathbb{R}$ *null infinity*.
- Diffeomorphism invariant.
- Naturally 'holographic'.



Massless amplitudes at null infinity

Amplitudes $\mathcal{A}(k_1, .., k_n) \leftrightarrow$ scattering of plane waves $\mathrm{e}^{ik_i \cdot x}$ with

null $k_i = \omega_i(1, \Re z_i, \Im z_i, 1 - |z_i|^2) \quad z_i \in \mathbb{C}, \quad \omega_i =$ frequency/energy.

- To approach $\mathscr{I}$, set $x^a = ut^a + rl^a$ with t^a unit timelike and l^a null

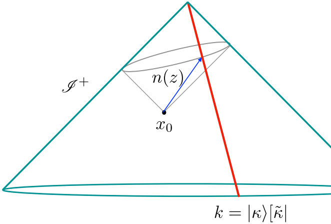
$$l^a = (1, , \Re z, \Im z, 1 - |z|^2).$$

- As $r \to \infty$ get coords

$$(u, z) \in \mathbb{R} \times S^2 = \mathscr{I}.$$

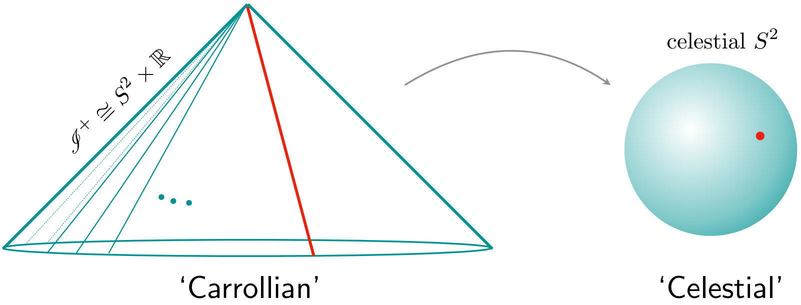
- Stationary phase gives

$$\lim_{r \to \infty} \mathrm{e}^{ik_i \cdot x} \sim \mathrm{e}^{i\omega_i u}\, \delta^2(z - z_i)\,.$$



Plane waves localize at pts of celestial sphere $= S^2$-factor of $\mathscr{I}$.

3 approaches to flat space holography



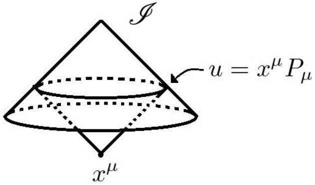
- **Carrollian.** Seeks *Carrollian CFT* with 'BMS' symmetries on $\mathscr{I}$ = null hypersurface. [Donnay, Fiorucci, Ruzziconi . . .]
- **Celestial.** Seeks ordinary CFT on celestial S^2: $u \in \mathbb{R}$-factor 'Mellin' transforms to Δ = dimension.

  [Strominger, Pasterski, Puhm, Ball, Kapec, Guevara, . . .]
- **Twistorial.** Seeks dual theory on asymptotic twistor space $\mathbb{PT} = \mathbb{CP}^3$, related to above by Penrose transform.

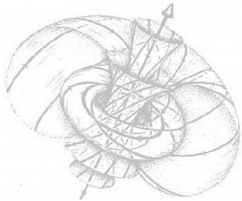
  [M., Adamo, Sharma, Costello, Paquette, Skinner, Bittleston, Zhang, Garner, . . .]

Prehistory: holography from null infinity, via light-cones

- **Newman:** '70's: tries to rebuild space-time from 'cuts' of $\mathscr{I}$.
- Yields instead '$\mathcal{H}$-space' a complex self-dual space-time.
- **Penrose:** $\rightsquigarrow$ asymptotic Twistor space $\mathcal{PT}$ & *nonlinear graviton*.



- **Idea:** Deform $\mathbb{PT} = \mathbb{CP}^3 - \mathbb{CP}^1 \rightsquigarrow \mathcal{PT}$ encoding data at $\mathscr{I}$.
- Embodies integrability of SD sector.
- $\rightsquigarrow$ hierarchies of hidden symmetries.



We will see Noether charges via twistor actions $\leftrightarrow$ Celestial Holography construction from soft OPEs of S-matrix.

Outline

1. Review of celestial symmetries.
2. Symmetries $=$ Twistor space diffeomorphisms.
3. $\leftrightarrow$ gravitons via Penrose's nonlinear graviton.
4. Twistor action for (SD) Einstein gravity.
5. Noether charges, and Hamiltonians/fluxes.
6. Recursion operators and hierarchies from BMS.

Bottom up input

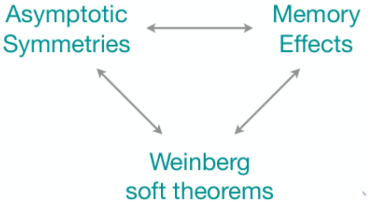
Postulate:

- amplitudes are correlators in some *celestial CFT*,

$$\mathcal{A}(k_1, k_2, \dots) = \langle \mathcal{O}(k_1)\mathcal{O}(k_2)\dots\rangle$$

- Transform the ω_i: Fourier $\rightsquigarrow$ Carrollian, Mellin $\rightsquigarrow$ celestial, $\frac{1}{2}$-Fourier/Penrose $\rightsquigarrow$ twistorial.
- Amplitudes give much data to work from $\rightsquigarrow$ new emergent structures!
- Infrared triangle:

[Strominger et/ al. 2013 on]



- New hierarchies of *celestial symmetries* $Lw_{1+\infty} = L\mathfrak{ham}(\mathbb{C}^2)$.

[Strominger, Ball, Guevara, Himwich, . . .]

Celestial symmetries

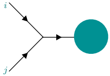
[Strominger, Guevara,Himwich, Pate,]

- Postulate amplitudes are correlators in a *celestial CFT*.

$$\mathcal{A}(k_1, k_2, \dots) = \langle \mathcal{O}(k_1)\mathcal{O}(k_2)\dots\rangle$$

- Let $k_i = \omega_i(1+|z_i|^2, \Re z_i, \Im z_i, 1-|z_i|^2)$, $z_i \in \mathbb{CP}^1$, ω_i energy

  soft expansion: $$\mathcal{O}(k) = \sum_p \omega^p H^p(z,\tilde{z})\,.$$



- Collinear limits $\rightsquigarrow$ *splitting functions*

$$\mathcal{M}(k_1,k_2,k_3,\dots) \xrightarrow{k_1\|k_2} \mathrm{Split}(k_1,k_2,k_1+k_2)\mathcal{M}(k_1+k_2,k_3,\dots)$$

- Splitting functions define OPE coefficients

$$H^{p_1}(z,\tilde{z}_1)H^{p_2}(0,\tilde{z}_2) \xrightarrow{z\to 0} \frac{C^{p_1p_2}_{p_3}}{z} H^{p_3}(0,\tilde{z}_1+\tilde{z}_2) + \dots$$

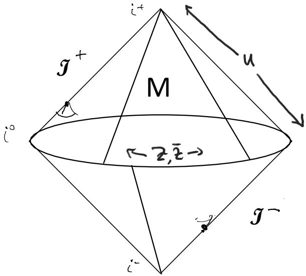
- Mode expansion $\rightsquigarrow Lw_{1+\infty} = L\mathfrak{ham}(\mathbb{C}^2)$:

$$[H^p_{m,a}, H^q_{n,b}] = (m(q-1)-n(p-1))H^{p+q-2}_{m+n,a+b}\,.$$

Data at null infinity $\mathscr{I}$

Asymptotically simple (M, g) has $\mathscr{I}^{\pm}$, all light rays meet both.

- Bondi coords: $(u = t - r, z = \frac{x+iy}{r-z}, \bar{z})$.
- At $\mathscr{I}$, $z \in \mathbb{C}$ stereographic coord. on S^2



- For curved g near $\mathscr{I}$ as $r \to \infty$

$$d\tilde{s}^2 = 2dudr - r^2 dzd\bar{z} + r(\sigma d\bar{z}^2 + \bar{\sigma} dz^2) + O(1)\,,$$

- $\sigma(u, z, \bar{z})$ = *asymptotic shear*; SD gravitational data at $\mathscr{I}$.

Realization as asymptotic charges

Friedel, Pranzetti & Raclariu, 2112.15573, Geiller 2403., Kmec, M., Ruzziconi, Yelleshpur-Srikant, 2407.04028

On $\mathscr{I} = \mathbb{R} \times S^2$ with coordinates $(u, z, \bar{z})$, introduce

- asymptotic shear (gravity data) $\leftrightarrow (\sigma(u, z, \bar{z}), \bar{\sigma}(u, z, \bar{z}))$,
- Weyl tensor NP components Ψ_n^0, $n = 0, \ldots, 4$

$$\Psi_4^0 = \ddot{\sigma}, \qquad \Psi_3^0 = \bar{\eth}\dot{\sigma},$$

- Asymptotic Bianchi identities

$$\partial_u \Psi_n^0 = \bar{\eth}\Psi_{n+1}^0 + (3-n)\bar{\sigma}\Psi_{n+2}^0, \qquad n = 0, 1, 2, 3\,.$$

- Generalise to recursion $\mathcal{Q}_s = \Psi_{2-s}^0$

$$\partial_u \mathcal{Q}_s = \bar{\eth}\mathcal{Q}_{s-1} + (s+1)\bar{\sigma}\mathcal{Q}_{s-2}, \qquad s = -1, 0, 1, 2 \ldots$$

- $\rightsquigarrow$ charges for $\tau_s(z, \bar{z})$ spin s, ($s = 0$ Bondi mass):

$$Q_s = \int_{S^2_{u=u_0}} d^2z\, \tau_s \mathcal{Q}_s + \ldots$$

- But guesswork, how do $Q_s \leftrightarrow H^p \ldots$? and its complicated:

The full formulae at $\mathscr{I}$

Friedel,Pranzetti, Raclariu][Geiller][Kmec,M., Ruzziconi, Yelleshpur]

$$Q_{-1} = \int_{\mathcal{S}} d^2 z\, \tau_{-1} \mathcal{Q}_{-1},$$

$$Q_0 = \int_{\mathcal{S}} d^2 z\, \tau_0 (\mathcal{Q}_0 - u\bar{\eth}\mathcal{Q}_{-1}),$$

$$Q_1 = \int_{\mathcal{S}} d^2 z\, \tau_1 \Big(\mathcal{Q}_1 - u\bar{\eth}\mathcal{Q}_0 + \frac{u^2}{2}\bar{\eth}^2 \mathcal{Q}_{-1} - 2\mathcal{Q}_{-1}\partial_u^{-1}\bar{\sigma} \Big),$$

$$Q_2 = \int_{\mathcal{S}} d^2 z\, \tau_2 \Big(\mathcal{Q}_2 - u\bar{\eth}\mathcal{Q}_1 + \frac{u^2}{2}\bar{\eth}\mathcal{Q}_0 - \frac{u^3}{6}\bar{\eth}^3 \mathcal{Q}_{-1}$$
$$- 3\mathcal{Q}_0\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}\bar{\sigma} + 2\bar{\eth}\left(\mathcal{Q}_{-1}\partial_u^{-1}(u\bar{\sigma}) \right) \Big),$$

$$Q_3 = \int_{\mathcal{S}} d^2 z\, \tau_3 \Big(\mathcal{Q}_3 - u\bar{\eth}\mathcal{Q}_2 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_1 - \frac{u^3}{6}\bar{\eth}^3 \mathcal{Q}_0 + \frac{u^4}{24}\bar{\eth}^4 \mathcal{Q}_{-1}$$
$$- 4\mathcal{Q}_1\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\left(\mathcal{Q}_0\partial_u^{-1}(u\bar{\sigma}) \right) + 4\bar{\eth}\mathcal{Q}_0\partial_u^{-2}\bar{\sigma} + 8\mathcal{Q}_{-1}\partial_u^{-1}(\bar{\sigma}\partial_u^{-1}\bar{\sigma})$$
$$- 3\bar{\eth}\left(\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}(u\bar{\sigma}) \right) - \bar{\eth}^2\left(\mathcal{Q}_{-1}\partial_u^{-1}(u^2\bar{\sigma}) \right) - 4\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-3}\bar{\sigma} \Big).$$

We give a top-down derivation from twistor space.

Twistor space as homogeneous space of $L\mathfrak{ham}(\mathbb{C}^2)$

Flat twistor space: $\mathbb{T} = \mathbb{C}^4$, projectively $\mathbb{PT}$ with hgs coords:

$$W^A = (\lambda_\alpha, \mu^{\dot\alpha}) \in \mathbb{T}, \quad \alpha = 0, 1, \quad W \sim aW\,, a \neq 0\,.$$

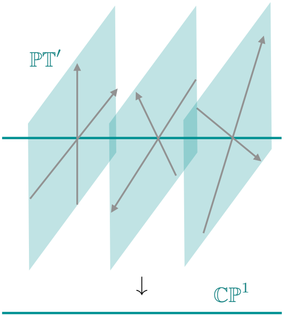
- Has projection $\mathbb{PT} - \mathbb{CP}^1_{\{\lambda_\alpha = 0\}}$ onto celestial sphere

$$p : \mathbb{PT} \to \mathbb{CP}^1\,, \qquad p(W) = \lambda_\alpha\,, \quad z = \lambda_1/\lambda_0\,.$$

- Poisson bracket: $$\{f, g\} = \varepsilon^{\dot\alpha\dot\beta} \frac{\partial f}{\partial \mu^{\dot\alpha}} \frac{\partial g}{\partial \mu^{\dot\beta}} = \left[\frac{\partial f}{\partial \mu}\, \frac{\partial g}{\partial \mu}\right].$$

- $L\mathfrak{ham}(\mathbb{C}^2)$ = Hamiltonian diffeomorphisms of $\mathbb{PT}$.
- Soft modes above are

$$H^p_{m,r} = \frac{(\mu^{\dot 0})^{p-m-1}(\mu^{\dot 1})^{p+m-1}}{\lambda_1^r \lambda_0^{2p-r-4}}\,.$$



Hamiltonians as gravitons

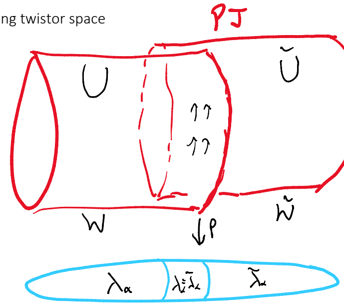
Theorem (Penrose 1976)

There is a $1:1$ *correspondence between:*

- *SD Ricci flat holomorphic metrics on regions in* $\mathbb{C}^4$*, and*
- *deformations* $P\mathcal{T}$ *of twistor space* $\mathbb{PT} - \mathbb{CP}^1$ *preserving fibration* $p : P\mathcal{T} \to \mathbb{CP}^1$ *and Poisson structure.*

- Cover $\mathbb{PT} - \mathbb{CP}^1$ by patches $U, \tilde{U}$ coords W and $\tilde{W}$ resp..
- Deform holomorphic gluing $\tilde{W} = \tilde{W}(W)$ on $U \cap \tilde{U}$:
- To preserve p and Poisson structure $\Rightarrow$ Hamiltonian H.
- expands in soft modes $H^p_{m,r}$.

Patching twistor space



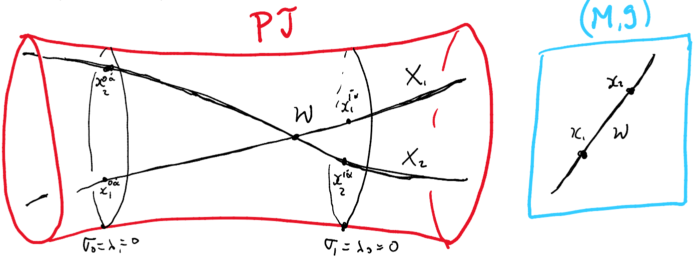
The SD space-time from holomorphic curves

Reconstruct self-dual space-time as

$$(M^4, g) = \{\text{Holomorphic degree-1 } \mathbb{CP}^1\text{s in } \mathbb{PT}\}\,.$$

- Parametrize the $\mathbb{CP}^1_x \subset \mathbb{PT}$ by coord z, $W^A = W^A(x, z)$

Correspondence with space-time



Dolbeault version: deform $\bar\partial$-operator with Poisson compatible Beltrami differential $\rightsquigarrow$ d-bar eq for $\mathbb{C}$-curves in deformed $\mathbb{PT}$:

$$\frac{\partial W^A}{\partial \bar z} d\bar z = \{\mathbf{h}, W^A\}|_{\mathbb{CP}^1_x}\,, \qquad \mathbf{h} \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2))\,.$$

SD Twistor action

Holomorphic Poisson Chern-Simons action [M. & Wolf 2007]

Dolbeault framework allows off-shell treatment:

- Gravitational data, SD: $\mathbf{h} \in \Omega^{0,1}(2)$, ASD: $\mathbf{g} \in \Omega^{0,1}(-6)$,

$$\mathbf{h}(aW) = a^2\mathbf{h}(W), \qquad \mathbf{g}(aW) = a^{-6}\mathbf{g}(W)\,, \qquad \mathbf{h} = \mathbf{h}_{\bar{A}} d\bar{W}^{\bar{A}} \text{ etc.}$$

- To guarantee compatibility with $\{,\}$, deform $\bar{\partial}$-operator by

$$\bar{\partial}_{\mathbf{h}} f(W, \bar{W}) := \bar{\partial}_0 f + \{\mathbf{h}, f\}$$

- Need integrability $\bar{\partial}_{\mathbf{h}}^2 = 0 \rightsquigarrow$ SD field equations

$$\bar{\partial}_0 \mathbf{h} + \frac{1}{2}\{\mathbf{h}, \mathbf{h}\} = 0\,, \qquad \bar{\partial}_{\mathbf{h}} \mathbf{g} = 0\,.$$

- Follow from action, with $D^3W = \langle W\, dW \wedge \ldots \wedge dW \rangle$,

$$S[\mathbf{g}, \mathbf{h}] := \int_{\mathbb{PT}} \mathbf{g} \wedge (\bar{\partial}\mathbf{h} + \frac{1}{2}\{\mathbf{h}, \mathbf{h}\}) \wedge D^3W\,.$$

So $[\mathbf{g}] \in H^1(\mathcal{PT}, \mathcal{O}(-6))$ and $\mathbf{h}$ a nonlinear $H^1(\mathcal{PT}, \mathcal{O}(2))$.

- local 'gauge' symmetries $= (\chi, \xi) \in (\mathcal{O}(-6), \mathcal{O}(2))$.

$$\delta(\mathbf{g}, \mathbf{h}) = (\bar{\partial}_{\mathbf{h}}\chi + \{\xi, \mathbf{g}\}, \bar{\partial}_{\mathbf{h}}\xi)$$

- $\xi \leftrightarrow$ generators of smooth local Poisson diffeos.
- $\chi \leftrightarrow$ gauge for $\mathbf{g} \in$ cohomology class.
- Action $\rightsquigarrow$ symplectic form

$$\Omega(\delta_1, \delta_2) = \int_{\Sigma^5} \delta_1\mathbf{g} \wedge \delta_2\mathbf{h} \wedge D^3W + (1 \leftrightarrow 2)\,.$$

- Gauge generated by Hamiltonians

$$Q_{\chi,\xi} = \int_{\Sigma^5} (\bar{\partial}_{\mathbf{h}}\xi \wedge \mathbf{g} + (\bar{\partial}_0\chi + \frac{1}{2}\{\mathbf{h}, \chi\}) \wedge \mathbf{h}) \wedge D^3W$$

- Compute Poisson brackets in gravitational phase space:

$$\{Q_{\chi_1,\xi_1}, Q_{\chi_2,\xi_2}\}_\Omega = Q_{\{\xi_1,\chi_2\}-\{\xi_2,\chi_1\},\{\xi_1,\xi_2\}}\,.$$

Algebra of charges; twistor space representation

- 'Large' gauge transformations are only symmetries if holomorphic at $\partial\Sigma^5 \subset \infty$', $\bar\partial_{\mathbf{h}}\xi = 0 = \bar\partial_{\mathbf{h}}\chi$.
- On-shell, Hamiltonians given by boundary charges

$$Q_{\chi,\xi} = \int_{\partial\Sigma^5} \xi\mathbf{g} + \chi\mathbf{h}\, D^3 W\,.$$

Here $\partial\Sigma^5 = \mathbb{CP}^1 \times \mathcal{S}$ for $M \supset \mathcal{S}$, some 2-surface at ∞.

- As above obtain gauge algebra

$$\{Q_{\chi_1,\xi_1}, Q_{\chi_2,\xi_2}\}_\Omega = Q_{\{\xi_1,\chi_2\}-\{\xi_2,\chi_1\},\{\xi_1,\xi_2\}}\,.$$

- Here to be symmetries, (χ,ξ) are holomorphic near $\partial\Sigma^5$.
- If global on $\partial\Sigma^5$, then $\xi =$ quadratic $\leftrightarrow$ Poincaré$^+$.
- $L\mathfrak{ham}(\mathbb{C}^2)$: take $\xi = H^p_{m,r} = \frac{(\mu^{\dot 0})^{p-m-1}(\mu^{\dot 1})^{p+m-1}}{\lambda_1^r\lambda_0^{2p-r-4}}$.
- χs $\rightsquigarrow$ dual $\tilde H$ s for $T^*L\mathfrak{ham}(\mathbb{C}^2)$, give ASD soft modes.

Asymptotic Twistor space and transform to $\mathscr{I}$

Sparling, Eastwood, Tod '81: Dolbeault version of Penrose's nonlinear graviton at $\mathscr{I}$.

- Project $\mathcal{T} \to \mathscr{I}$ with fibres $\mathbb{CP}^1_{(u,z,\bar{z})} \ni q$ given by

$$z = \frac{\lambda_1}{\lambda_0}\,, \qquad \mu^{\dot\alpha} = uT^{\alpha\dot\alpha}\lambda_\alpha + q\bar\lambda^{\dot\alpha}\,, \qquad T^{\alpha\dot\alpha} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- Asymptotic shear $\sigma d\bar{z}$ pulls back to $\mathcal{T}$ to define

$$\mathbf{h} = d\bar{z}\int^u du'\sigma(u', z, \bar{z}) = h(\mu^{\dot\alpha}\bar\lambda_{\dot\alpha}, \lambda, \bar\lambda)[\bar\lambda d\bar\lambda] \in \Omega^{0,1}(2)\,.$$

  Gives deformed $\bar\partial$-operator on $\mathcal{T}$ as $\bar\partial_{\mathbf{h}} f := \bar\partial_0 f + \{\mathbf{h}, f\}$.

- Charge aspects $\Psi^0_n = \mathcal{Q}_{2-n}$ for $n = 0, .., 3, \leftrightarrow \mathbf{g}$ by

$$\mathcal{Q}_s = \int_{\mathbb{CP}^1_{(u,z,\bar{z})}} q^{s+2}\mathbf{g} \wedge \mathrm{d}q$$

- Solves asymptotic Bianchi identities & recurrence relation

$$\partial_u \mathcal{Q}_s = \bar\eth\mathcal{Q}_{s-1} + (s+1)\bar\sigma\mathcal{Q}_{s-2}, \qquad s = -1, 0, 1, 2$$

Extension to $s > 1 \leftrightarrow L\mathfrak{ham}(\mathbb{C}^2)$ built from Poincaré$^+$ via $\leftrightarrow \times q$.

Gauge fixing the Poisson diffeos

Our ξ generates Poisson diffeos of $P\mathcal{T}$ that destroys gauge fixing of $\mathbf{h}$ that relates $\partial_u \mathbf{h}$ to $\sigma(u, z, \bar{z})$. We fix this as follows:

- We need the gauge fixing $\partial_q \mathbf{h} = 0$ which gives

$$\partial_q (q\partial_u - \bar{\eth} + \bar{\sigma}\partial_q)\xi = 0 \,.$$

- Solve by ansatze $\xi_s = \sum_{n=0}^{s+1} \xi_{s,n} q^n$ and recurrence

$$\partial_u \xi_{s,n-1} - \bar{\eth}\xi_{s,n} + (n+1)\bar{\sigma}\xi_{s,n+1} = 0 \,, \qquad \xi_{s,s+1} := \tau_s(\lambda, \bar{\lambda}) \,.$$

- Start with $\tilde{\xi}_s \leftrightarrow \tau_s(\lambda, \bar{\lambda})$ spin s on S^2, poly degree $s+1$ in q

$$\tilde{\xi}_s = \frac{\mu^{\dot{\alpha}_1} \dots \mu^{\dot{\alpha}_{s+1}}}{(s+1)!} \frac{\partial^{s+1} \tau_s}{\partial \bar{\lambda}^{\dot{\alpha}_1} \dots \partial \bar{\lambda}^{\dot{\alpha}_{s+1}}} = \sum_{n=-1}^{s} \frac{u^{s-n} q^{n+1}}{(s-n)!} \bar{\eth}^{s-n} \tau_s \,,$$

- 'Wedge' condition $\Leftrightarrow \bar{\eth}^{s+2}\tau_s = 0 \Leftrightarrow \tilde{\xi}_s$ holomorphic.
- Then find gauge fixed $\xi_s = \tilde{\xi}_s + f_s$ solving recursively for f_s.

But, ξ_s now *field dependent* $\rightsquigarrow$ nonintegrable (non-Hamiltonian)!

The charges from twistor space to $\mathscr{I}$

Define surface charge integrals $Q_s := \int_{\partial\Sigma^5} \xi_s \, \mathbf{g} \wedge D^3 Z$ giving:

$$
\begin{aligned}
Q_{-1} &= \int_{\mathcal{S}} d^2 z \, \tau_{-1} \mathcal{Q}_{-1}, \\
Q_0 &= \int_{\mathcal{S}} d^2 z \, \tau_0 (\mathcal{Q}_0 - u\bar{\eth}\mathcal{Q}_{-1}), \\
Q_1 &= \int_{\mathcal{S}} d^2 z \, \tau_1 \Big(\mathcal{Q}_1 - u\bar{\eth}\mathcal{Q}_0 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_{-1} - 2\mathcal{Q}_{-1}\partial_u^{-1}\bar{\sigma} \Big), \\
Q_2 &= \int_{\mathcal{S}} d^2 z \, \tau_2 \Big(\mathcal{Q}_2 - u\bar{\eth}\mathcal{Q}_1 + \frac{u^2}{2}\bar{\eth}\mathcal{Q}_0 - \frac{u^3}{6}\bar{\eth}^3\mathcal{Q}_{-1} \\
&\qquad - 3\mathcal{Q}_0\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}\bar{\sigma} + 2\bar{\eth}\left(\mathcal{Q}_{-1}\partial_u^{-1}(u\bar{\sigma}) \right) \Big), \\
Q_3 &= \int_{\mathcal{S}} d^2 z \, \tau_3 \Big(\mathcal{Q}_3 - u\bar{\eth}\mathcal{Q}_2 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_1 - \frac{u^3}{6}\bar{\eth}^3\mathcal{Q}_0 + \frac{u^4}{24}\bar{\eth}^4\mathcal{Q}_{-1} \\
&\quad - 4\mathcal{Q}_1\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\left(\mathcal{Q}_0\partial_u^{-1}(u\bar{\sigma}) \right) + 4\bar{\eth}\mathcal{Q}_0\partial_u^{-2}\bar{\sigma} + 8\mathcal{Q}_{-1}\partial_u^{-1}(\bar{\sigma}\partial_u^{-1}\bar{\sigma}) \\
&\quad - 3\bar{\eth}\left(\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}(u\bar{\sigma}) \right) - \bar{\eth}^2\left(\mathcal{Q}_{-1}\partial_u^{-1}(u^2\bar{\sigma}) \right) - 4\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-3}\bar{\sigma} \Big).
\end{aligned}
$$

Conclusions & discussion

Conclusions:

- $L\mathfrak{ham}(\mathbb{C}^2) \leftrightarrow$ geometric diffeo algebra on $\mathbb{PT}$
- Penrose nonlinear graviton: $\{$SD gravitons$\} = \{L\mathfrak{ham}(\mathbb{C}^2)\}$.
- $\rightsquigarrow$ Noether charges via Twistor action.
- Penrose transform from asymptotic twistor space to $\mathscr{I} \rightsquigarrow$ charge aspects, correcting [Friedel, Pranzetti, Raclariu][Geiller].
- Integrability of SD & MHV sector $\rightsquigarrow$ Recursion for hierarchy from BMS transitive over gravity phase space SDYM [M. & Woodhouse 1996] and SD gravity [Dunajski & M. 2001/2].

Where does take us?

- $\leftrightarrow$ 3d analogue of Virasoro for twistorial 'celestial CFTs'.
- Are these symmetries of S-matrix beyond MHV?
- Vertex operator realization gives action on higher MHV worldsheet amplitude formulae. cf Adamo, M., Sharma 2021.

Thank You!

Versions with cosmological constant

- Ward's Λ extension of Penrose's nonlinear graviton underpins Taylor-Zhu Λ-extension of $L\mathfrak{ham}(\mathbb{C}^2)$ at $\Lambda = 0$.
  [Bittleston, Bogna, Heuveline, Kmec,M., Skinner, 2403.18011].
- For $\Lambda \neq 0$ use 'Heaven on earth' construction [Lebrun 1983].
- Analogue of Virasoro for honest integrable CFT_3s!
- Use full twistor actions beyond SD sector cf. [M.,Boels, Sharma, Skinner].
- Connections with conventional AdS_4/CFT_3 e.g., ABJM?