LQG Effects on Forming a Black Hole: in Collaboration with Jurek Lewandowski

Yongge Ma

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Jerzy Lewandowski Memorial Conference, Sept. 19, 2025

Based on the joint work with Jerzy Lewandowski, Jinsong Yang and Cong Zhang

Outline

In collaboration with BNU

- Jurek's Visits to BNU
  1. 22 Nov. to 5 Dec., 2011
  2. 31 May to 22 June, 2014
  3. 16 to 31 July, 2017
  4. 23 Nov. to 10 Dec., 2018
  5. 13 to 17 May, 2019
  6. 14 to 30 July, 2019

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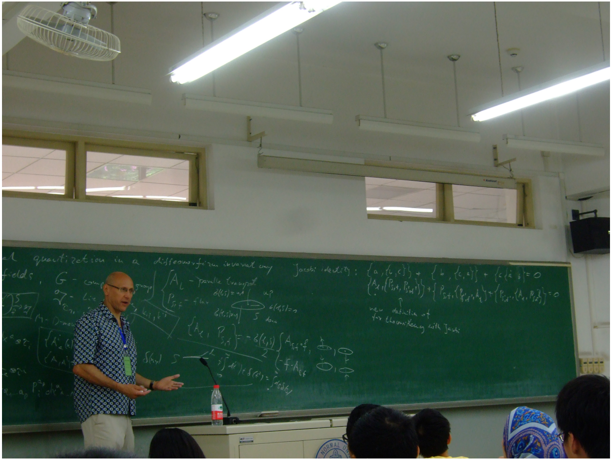
- SHENG 1 - Polish-Chinese Funding (2019-2022):
  Title: Dynamics and Extensions of Loop Quantum Gravity
  Participants: Jerzy Lewandowski, Yongge Ma,
  Wojciech Kaminski, Andrzej Okolow, Jinsong Yang, Chun-Yun Lin, Cong Zhang, etc.

Jurek's Lecture at BNU



Jurek's Lectures in China

1. Loops'09 Conference (2 to 7 Aug. 2009)
   Plenary talk: Spin Foams Generalized to All the States of LQG
2. 2nd BNU International Summer School on GR and QG (5 to 18 Aug, 2012)
   Short course: Framework of loop quantum gravity
3. 3rd BNU International Summer School on GR and QG (7 to 20 Aug, 2016)
   Short course: Canonical loop quantum gravity
4. Annual Conference of Chinese Society on Gravitation and Relativistic Astrophysics (14 to 19 July 2019)
   Plenary talk: Canonical and covariant LQG

Ideas of Loop Quantum Gravity

- ⋆ Loop Quantum Gravity inherits the basic idea of Einstein that gravity is fundamentally spacetime geometry.
  Hence the theory of quantum gravity is a quantum theory of spacetime geometry with background independence.
- ⋆ The choice of the algebra of field functions to be quantized: The holonomies of the gravitational connection and the electric flux:

$$h_e(A) = \mathcal{P}\exp\int_e A_a, \qquad E(S,f) := \int_S \epsilon_{abc} E^a_i f^i$$

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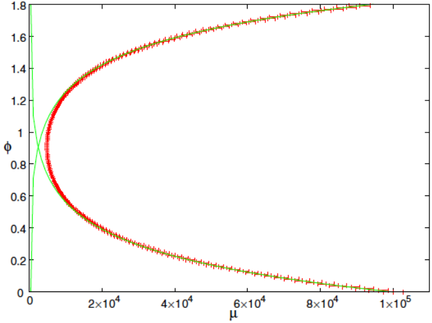
- The idea and technique of LQG is successfully carried out in the symmetry-reduced models, such as loop quantum cosmology and loop quantum black holes.

Big Bang Singularity Resolution

- Big bang singularity resolution in LQC [Ashtekar, Powlowski, Singh, PRL (2006); Ding, YM, Yang, PRL (2009); Yang, Ding, YM, PLB (2009); Assanioussi, Dapor, Liegener, Pawlowski, PRL (2018)]



Quantum Kruskal Black Holes

[Ashtekar, Olmedo, Singh, PRL (2018)]

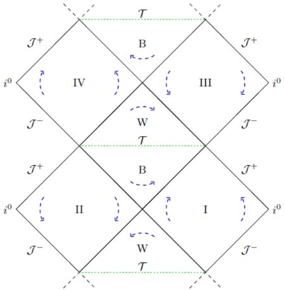


FIG. 1: The Penrose digram of the extended Kruskal space-time. The central diamond $B \cup W$ is the 'interior', containing the trapped region B and an anti-trapped region W, separated by the transition surface $\mathcal{T}$ that replaces the classical singularity. I, II, III and IV are asymptotic regions and the arrows represent the translational Killing vector $X^a \partial_a \equiv \partial/\partial x$.

What is the QG effect on forming a BH

- Could the BH to white hole transition actually happen in the matter collapsing procedure to form a BH?
- In classical GR, although a white hole appears in the Kruskal extension of Schwarzschild spacetime, it is not the case in the Oppenheimer-Snyder model, which depicts the collapse of the pressureless homogenous dust coupled to the FRW metric.

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- What would happen if we considered the quantum gravity effects in the Oppenheimer-Snyder model?

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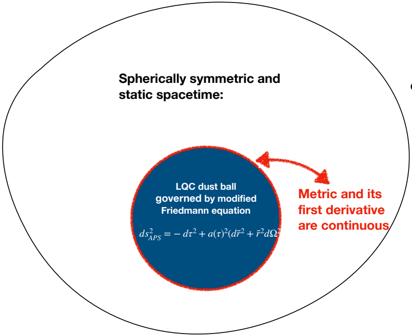
- What would happen if we considered the quantum gravity effects in the Oppenheimer-Snyder model?
- If the picture became different from that of classical theory, is there any observational effect?

BH to WH transition through LQC

[Lewandowski, YM, Yang, Zhang, PRL (2023)]

BH from LQC matter collapsing



$$ds^2_{\rm MS} = -(1 - F(r))dt^2 + (1 - G(r))^{-1}dr^2 + r^2 d\Omega^2$$

$$F(r) = G(r) = \frac{2GM}{r} - \frac{\alpha G^2 M^2}{r^4}$$

$$\alpha = 16\sqrt{3}\pi\gamma^3\ell_p^2$$

Modified Friedmann equation by APS dynamics:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \quad \rho = \frac{M}{\frac{4}{3}\pi\tilde{r}_0^3 a^3}$$

Gluing LQC with a BH

- The particles of the dust in the APS cosmological spacetime are the geodesics satisfying $\tilde{r}, \theta, \phi = \mathrm{const.}$
- Let the dust ball surface be given by $\tilde{r} = \tilde{r}_0$ and $\tau \mapsto (t(\tau), r(\tau), \theta, \phi)$ be a radial geodesic in the spherically symmetric static spacetime, with τ being the proper time.
- We glue the two spacetimes by the identification $(\tau, \tilde{r}_0, \theta, \phi) \sim (t(\tau), r(\tau), \theta, \phi)$, such that the induced metrics and the extrinsic curvatures of both sides are equal on the gluing surface.

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- It turns out that the junction condition uniquely determines the functions $F(r)$ and $G(r)$.

Global Structure of the Spacetime

- The vacuum spherically symmetric metric is obtained as

$$ds^2_{\mathrm{MS}} = -\left(1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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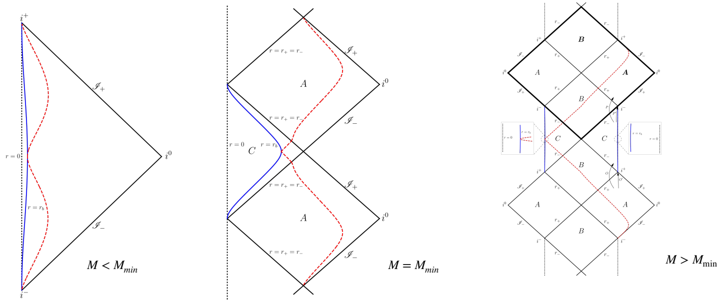
- The global structure of the spacetime determined by this metric depends on the number of roots of $1 - F(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$.
- For $M < M_{\mathrm{min}} := \frac{4}{3\sqrt{3}G}\sqrt{\alpha}$, it has no real root, implying that no horizon will be formed.

Three Different Cases

[Lewandowski, YM, Yang, Zhang, PRL (2023)]

<u>Existence of a minimal BH mass:</u> $M_{\text{min}} := \frac{4}{3\sqrt{3}G}\sqrt{\alpha}$



Quantum Swiss Cheese Model

[Lewandowski, YM, Yang, Zhang, PRL (2023)]

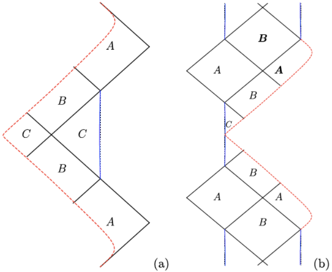
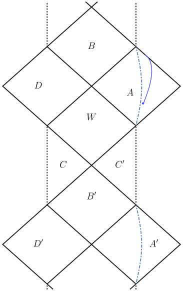


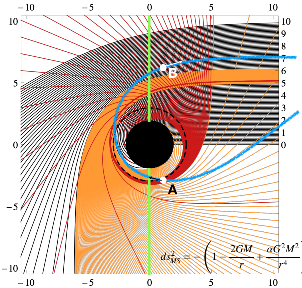
FIG. 2. (a) The piece outside the collapsing dust of the Penrose diagram containing the collapsing dust, for $1/2 < \beta < 1$. (b) The piece inside the LQC dust of the swiss cheese diagram. The A region labeled by the bold $\boldsymbol{A}$ is the asymptotically flat region of the current universe in the qSC model, and the one labeled by the bold $\boldsymbol{B}$ is the trapped region accessible for observers in the current universe.

Image of Quantum Modified Schwarzschild BH

[Yang, YM, Zhang, EPJC (2023)]

QG effects on BH image





$$I^{\text{obs}} = \sum_m f^2 I|_{r=r_m(b)}$$

$$ds^2_{MS} = -\left(1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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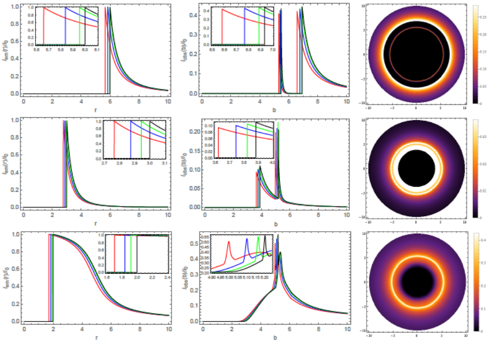
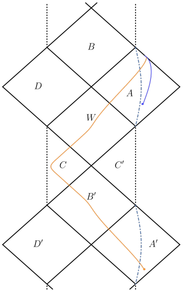


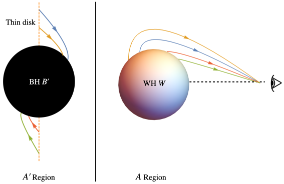
FIG. 6. The observational appearances of the thin disk near the BHs with the three different profiles: In each row, the first two panels show the emission intensity I_{em}/I_0 and observational intensity I_{obs}/I_0, normalized to the maximum value I_0, of a thin disk near the quantum-corrected BHs, corresponding to $k = -1$ (red), $k = 0$ (blue) and $k = +1$ (green), compared to those of the Schwarzschild BH (black), and the third panel depicts the density plot of I_{obs}/I_0 of a thin disk near the quantum-corrected BH with $k = 0$. The parameters are $R_s = 2$, $\gamma = 1$, $\Delta = 0.25$, and $\bar{r}_0 = 0.5$.

Image of the BH Companion of WH

[Zhang, YM, Yang, PRD (2023)]

BH image to probe QG





$$\left(\frac{\mathrm{d}u}{\mathrm{d}\phi}\right)^2 = -\alpha M^2 u^6 + 2Mu^3 - u^2 + \frac{1}{b^2} \equiv G(u,b), \quad (3)$$

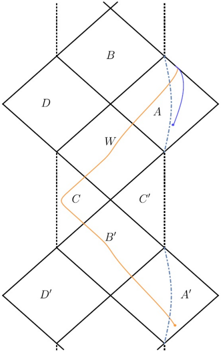
$$\frac{\mathrm{d}v_\pm}{\mathrm{d}u} = \frac{\pm b}{\sqrt{G(u,b)}(1+b\sqrt{G(u,b)})}, \quad (4)$$

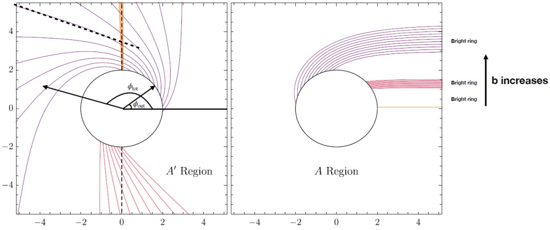
where $u \equiv 1/r$, $\phi \in (0,\infty)$ is the azimuthal angle in the orbit plane, and b, the impact factor given by the ratio of the angular momentum and energy, is a constant of motion and chosen to be positive. Note that the affine

Image of the BH Companion of WH

[Zhang, YM, Yang, PRD (2023)]

BH Image encoding QG information



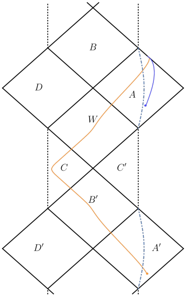


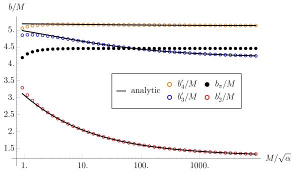
Critical rays: $\phi_{\rm tot}(b_n) = \pi/2 + n\pi$, **marginal line intersecting the disk;**
$\phi_{\rm out}(b_n') = \pi/2 + n\pi$, **marginal line NOT intersecting the disk;**
$\Delta\phi(b_\pi) = \pi$, **marginal line always intersecting the disk;**

Rays with $b_n \le b \le b_n'$ **and** $b < b_\pi$ **will be lighten by the disk, and contribute the bright rings**

Image of the BH Companion of WH

[Zhang, YM, Yang, PRD (2023)]



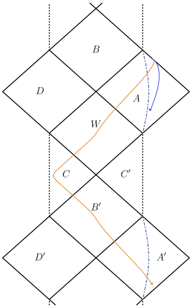


For large BH, there will be at least three bright rings.

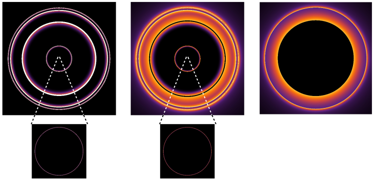
The intervals $(b_n/M, b'_n/M)$ to specify the bright rings as $M \to \infty$ can be obtained by applying Eq. (5) and ignoring higher order terms of $\sqrt{\alpha}/M$. The limits of $(b_n/M, b'_n/M)$ are $(0.0427, 0.0445)$, $(0.9242, 1.2620)$, $(2.7927, 4.1766)$, $(4.3175, 5.1285)$ and $(4.9648, 5.1931)$ for $n = 1, 2, 3, 4$ and 5 respectively. Moreover, the value of b_π/M as $M \to \infty$ is solved as 4.4573. Consequently, for sufficiently large M, we have $b'_3 < b_\pi < b'_4$. Therefore, there are at least three discrete bright rings in the BH image for a massive BH. These rings locate at $(b_n + b'_n)/(2M) \approx 0.04,\ 1.1$ and 3.5 respectively with widths $\Delta b/M \approx 0.002,\ 0.3$ and 1.4. The above results of analytical calculations are confirmed by numerical computation in their scope of application as shown in Fig. 4. It is also illustrated that b_π becomes smaller than b'_3 for small M. Hence, there would be two distinguishable bright rings in this case.

Image of the BH Companions

[Zhang, YM, Yang, PRD (2023)]



BH image to probe QG



$$\frac{\phi_{\rm tot}(\lambda M)}{2} = \int_0^\infty \frac{{\rm d}y}{\sqrt{2y^3 - y^2 + \lambda^{-2}}} - \frac{\sqrt[6]{4\alpha\pi^3}\,\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{1}{3}\right)\sqrt[3]{M}} + O(\alpha^{1/3}M^{-2/3}),$$

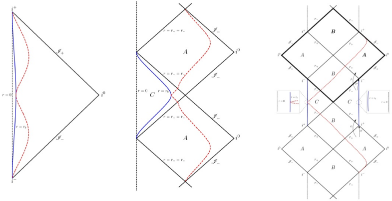
$$\Delta\phi(\lambda M) = \int_0^{\frac{1}{2}} \frac{{\rm d}y}{\sqrt{2y^3 - y^2 + \lambda^{-2}}} + O(\sqrt{\alpha}M^{-1}),$$

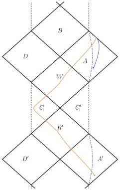
Note that the leading orders contributions to $\phi_{\rm tot}(\lambda M)/2$ and $\Delta\phi(\lambda M)$ coincides with the changes of the azimuthal angles for a light traveling from $r = \infty$ to the singularity and to the horizon in the Schwarzschild spacetime, respectively [39], since the quantum spacetime takes the Schwarzschild one as its classical limit.

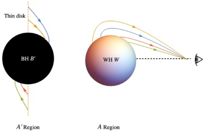
Summary of Previous Results

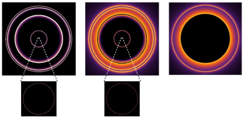
$$ds^2_{\rm MS} = -(1-F(r))dt^2 + (1-G(r))^{-1}dr^2 + r^2 d\Omega^2$$

$$F(r) = G(r) = \frac{2GM}{r} - \frac{\alpha G^2 M^2}{r^4} \qquad \alpha = 16\sqrt{3}\pi\gamma^3\ell_p^2$$



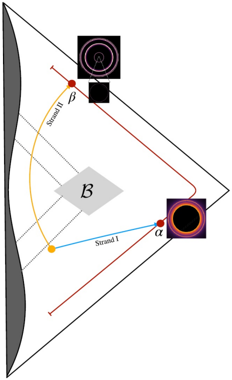






Discussion

- The spacetime proposed by [Han, Rovelli, Soltani, 2023] and its implication for the BH image:



!Thanks!