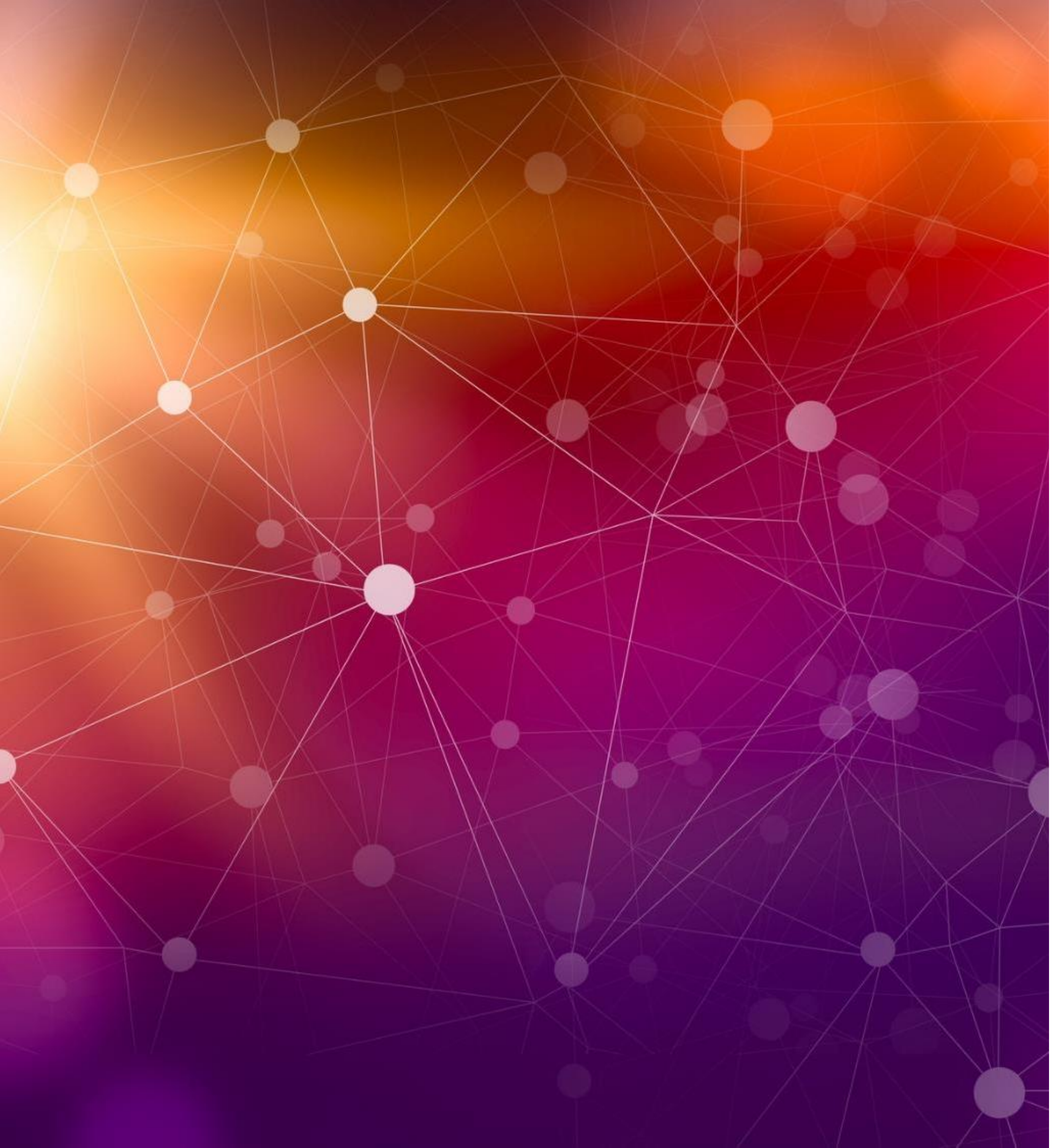

From Corner Proposal to Emergence of the Area Law

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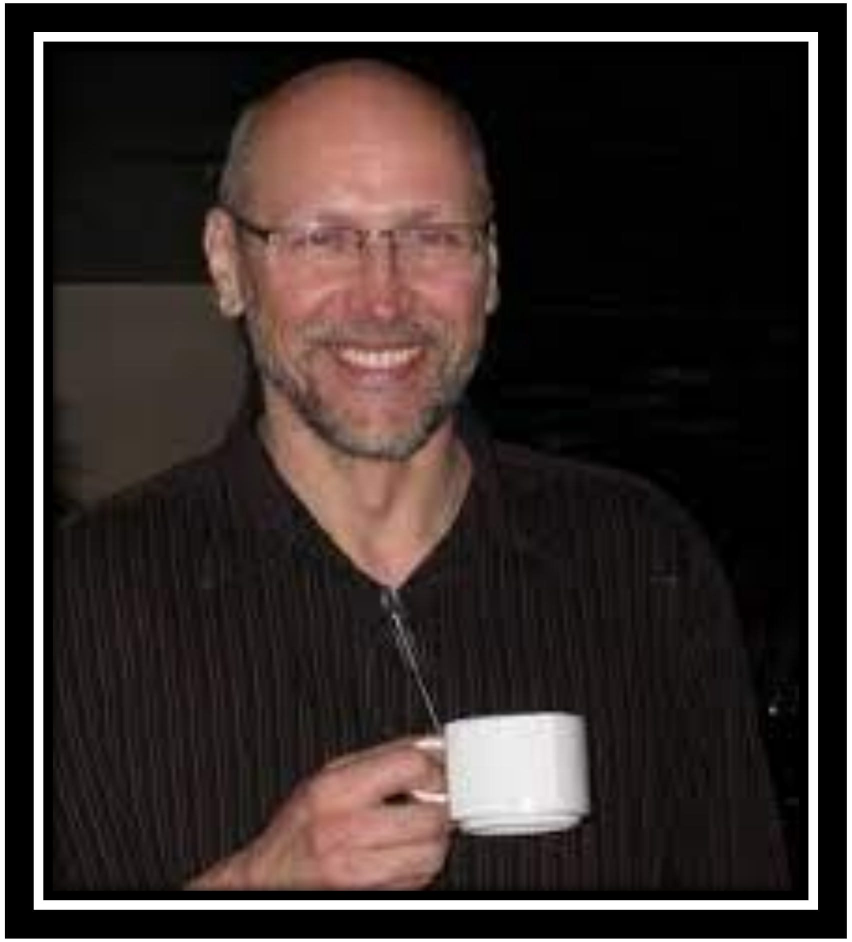
Based on works with Luca Ciambelli and Ludovic Varrin

Dedicated to Jurek Lewandowski



Motivations

- **AdS/CFT**:
  - Originally started as a conjecture, AdS/CFT correspondence proposes that string theory on the manifold $\mathrm{Ad}S^5 \times S^5$ is equivalent to CFT on its boundary, the 4D Minkowski space. It was soon realized that the bulk/boundary correspondence has far broader applications. This insight aligned with the holographic proposal put forth by ’t Hooft and Susskind.

- **Corner Conjecture**
  - The core idea of the **Corner Conjecture** is that the gravitational physics within a bounded region of spacetime is encoded on its codimension-2 boundary surface—the *corner*. In this sense, the corner is where the hologram resides.

On the menu

- Noether Theorem and codimension-2 surface (corner) charges
- Universal Corner Algebra and quantum corner conjecture.
- Quantum Corner Algebra in 2D.
- Gluing the segments.
- Entanglement entropy and the area law.

Covariant phase space method

- Consider the action

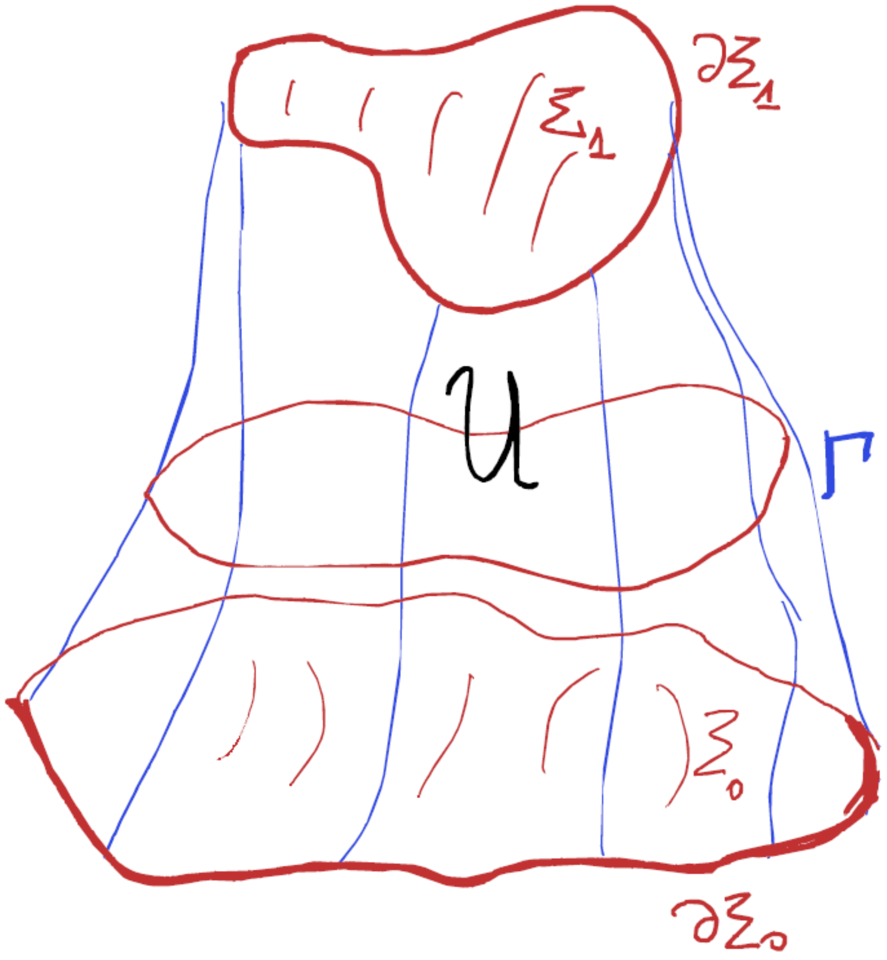
$$S = \int_{\mathcal{U}} \mathscr{L} + \int_{\partial \mathcal{U}} \ell$$

- Variation of the Lagrangian can be written as

$$\delta \mathscr{L} = (EOM)\delta\varphi + d\theta(\varphi, \delta\varphi)\,, \qquad \Theta = \int_{\Sigma} \theta$$

- The (3,1)-form θ is called the pre-symplectic potential.
- The associated pre-symplectic form

$$\Omega = \delta\Theta = \int_{\Sigma} \delta\theta$$



Corners and diffeomorphism charges

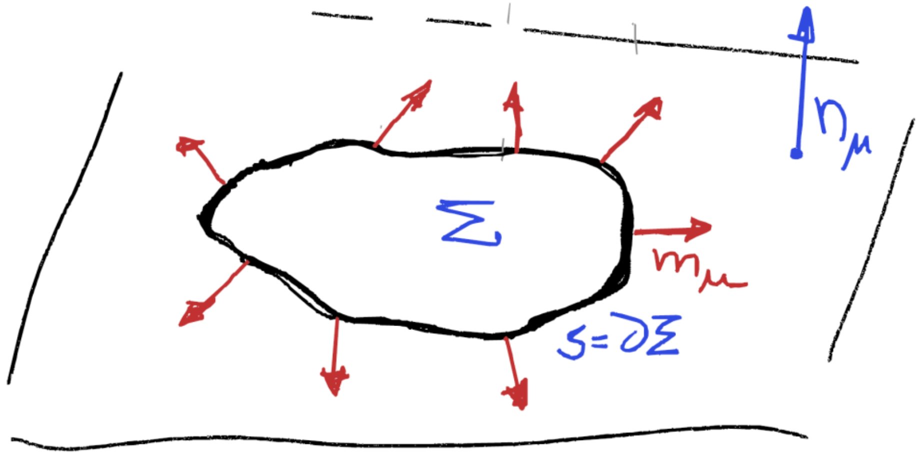
- One defines the conserved Noether current $\mathcal{J}[\xi]$ associated with the diffeomorphism generated by the vector field ξ

$$\mathscr{J}[\xi] = I_{V_\xi}\theta - i_\xi \mathscr{L} + \cdots \,,$$
$$d\mathscr{J}[\xi] = 0 \Longrightarrow \mathscr{J}[\xi] = d\mathscr{H}[\xi]$$

- The Noether current $\mathcal{J}[\xi]$ is a (d-1) form and integrating it on the codimesion-1 surface Σ, we find that the integral reduces to the integral over the codimension-2 boundary S, called the corner.

- In metric gravity the Noether charges are Komar integral charges

$$\mathscr{H}[\xi] = \int_S d\Sigma_{\alpha\beta}\sqrt{\sigma}\nabla^\alpha \xi^\beta$$



D. Harlow & J-Q. Wu 1906.08616 [hep-th]
V. Chandrasekaran et. al. 2111.11974 [gr-qc]
L. Ciambelli, 2212.13644 [hep-th]

Universal Corner SymmetryAlgebra

- Poisson brackets of charges form a representation of algebra of the associated symmetries. In the case of gravity, the most general corner algebra takes the form

$$\mathfrak{ucs} = \mathrm{Diff}(S) \oplus (\mathrm{SL}(2,\mathbb{R})^S \oplus (\mathbb{R}^2)^S)$$

- Expressed in terms of the metric, the charges are

$$\mathscr{H}[\xi] = \int_S d^2x \left(\xi^a_{(1)b}(x) N^b_a + \xi^j_{(0)}(x) b_j + \xi^a_{(0)}(x) p_a \right)$$

SL(2,R)

Diff

R^2

$$N^b_a = \sqrt{-\det h^{(0)}}\, h^{bc}_{(0)} \epsilon_{ca}$$

$$b_j = -N^b{}_a a^a_{(1)jb}$$

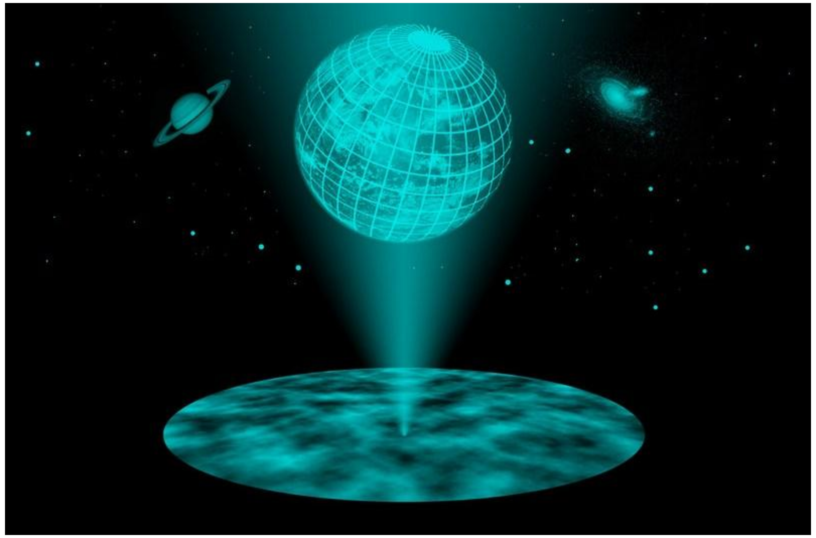
$$p_d = \frac{1}{2} N^a{}_c h^{cb}_{(0)} \left(h^{(1)}_{dba} - h^{(1)}_{dab} \right)$$

$$\mathrm{d}s^2 = h_{ab}(z,x) n^a n^b + \gamma_{ij}(z,x) \mathrm{d}x^i \, \mathrm{d}x^j$$

$$n^a = \mathrm{d}z^a - a^a_j(z,x) \mathrm{d}x^j$$

The Quantum Corner Conjecture

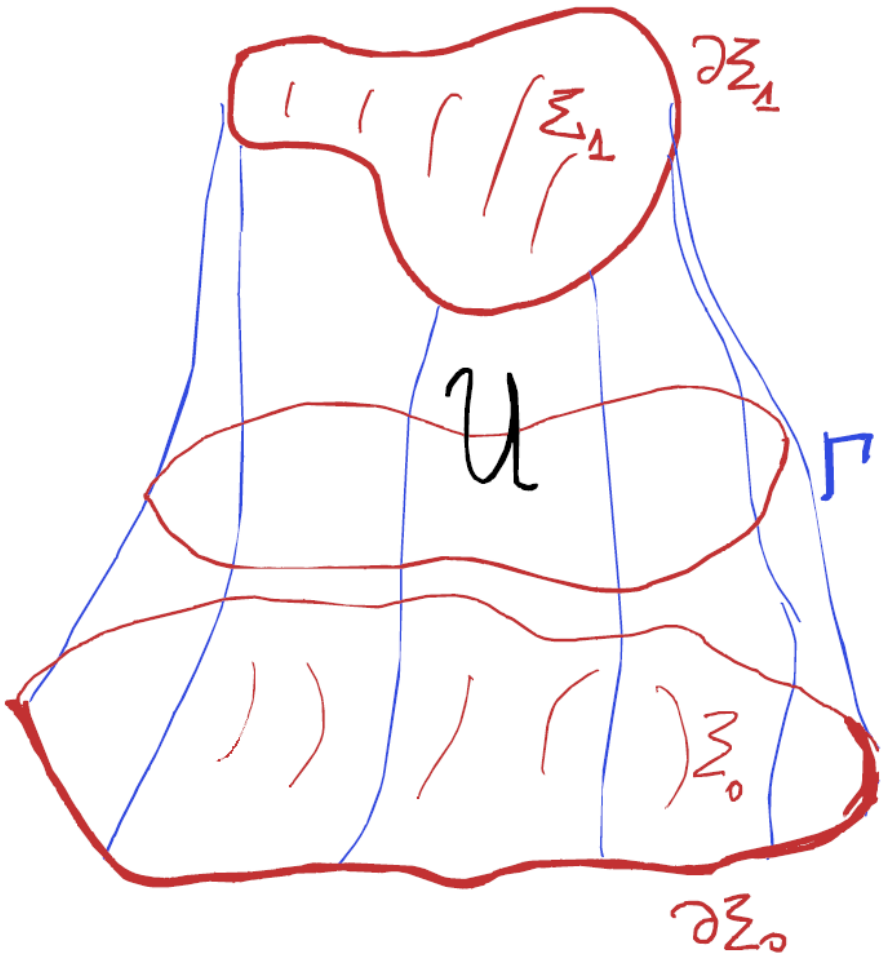
$$\mathfrak{ucs} = \mathrm{Diff}(S) \oplus (\mathrm{SL}(2,\mathbb{R})^S \oplus (\mathbb{R}^2)^S)$$



Holography organizes Quantum Gravity.

- *It is said that particle physics is a representation theory of Poincare group.*
- *The universality of the $\mathfrak{ucs}$ algebra suggests that it similarly encapsulates the fundamental kinematical structure of gravity.*
- *In analogy with representation theory of Poincare group in QFT, we conjecture that the representation theory of corner algebra organizes the Hilbert space of Quantum Gravity.*

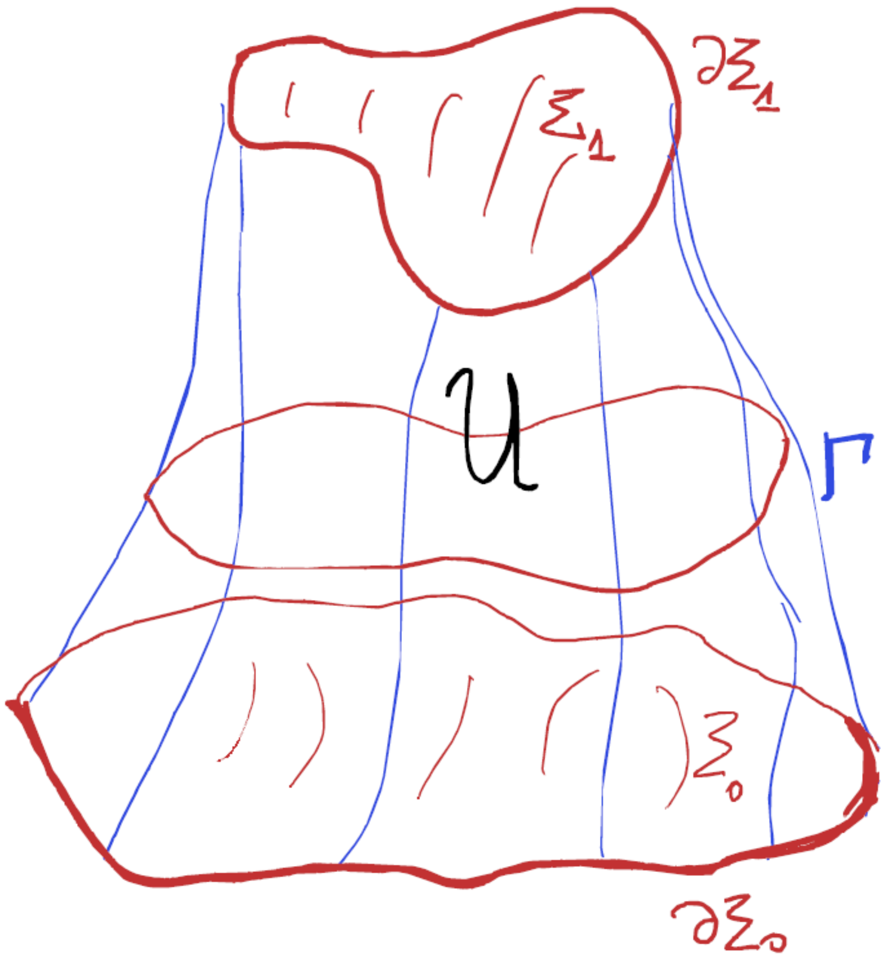
Diffeomorphisms



Diffeomorphisms

↓

Noether charges



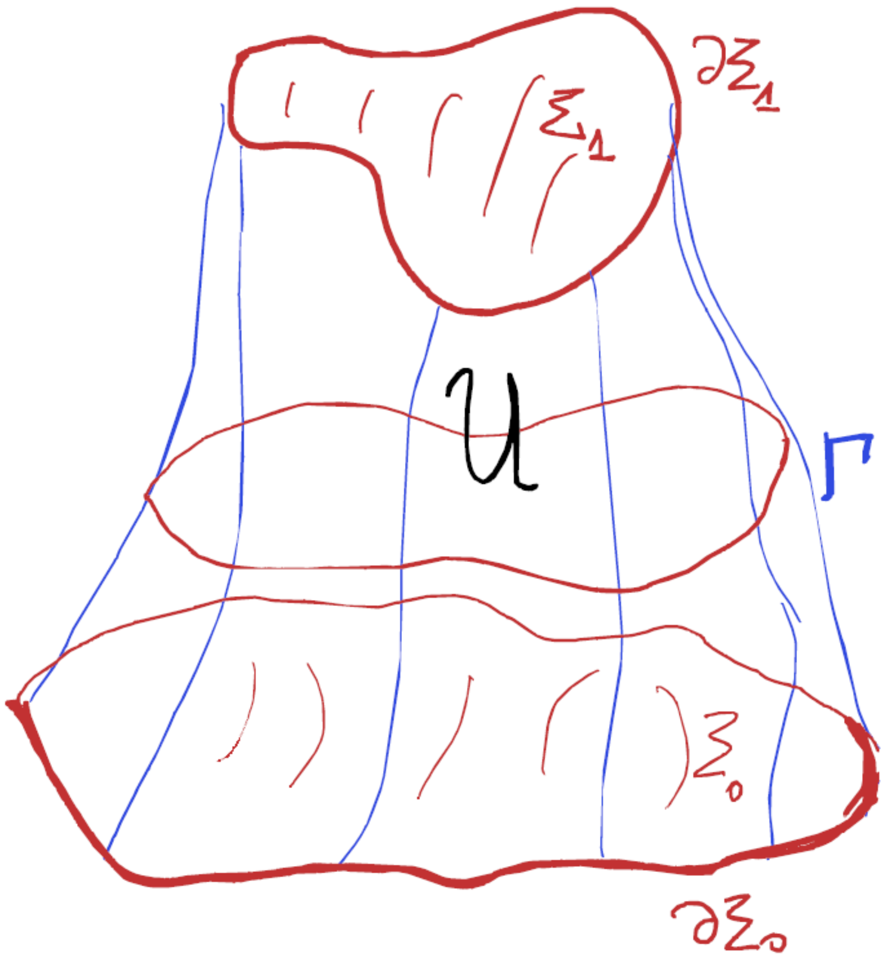
Diffeomorphisms

↓

Corner charges

↓

Universal Corner Algebra



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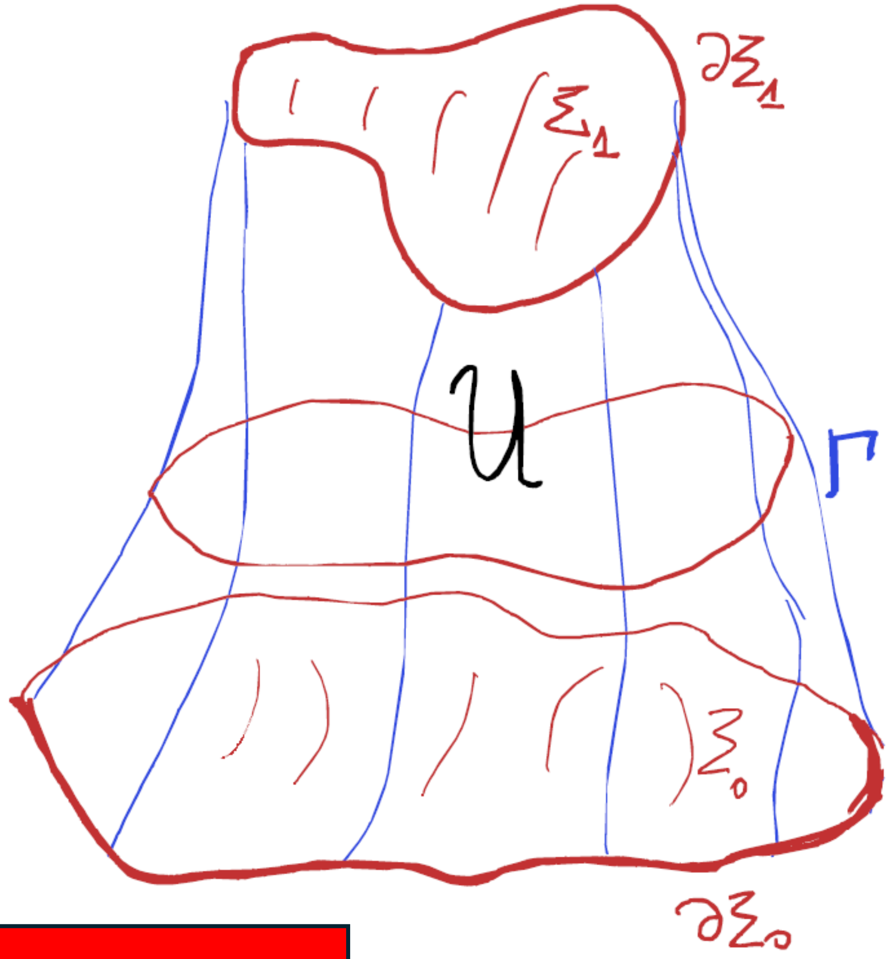
Universal Corner Algebra

classical

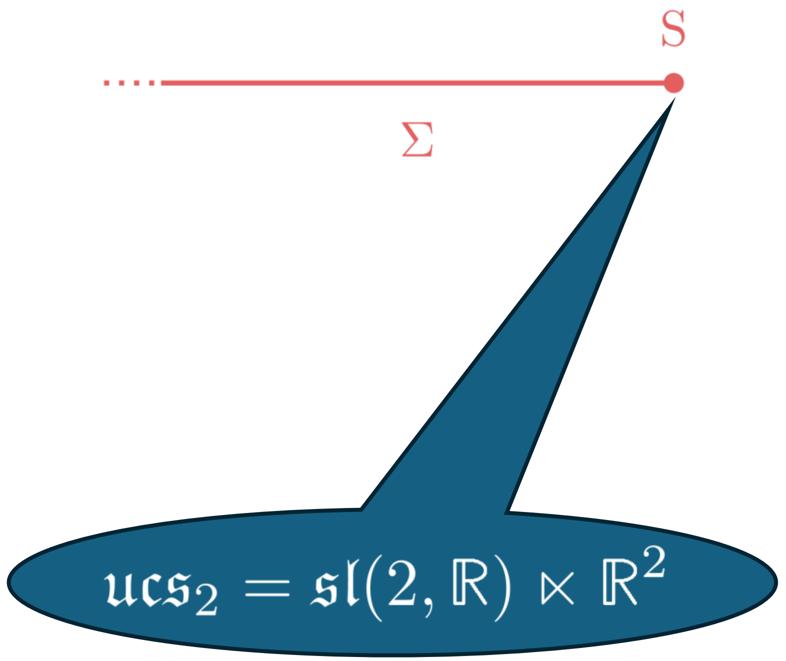
quantum

Representation theory → ……



The toy model: ECS_2 *

- In D=4 the representation theory of ECS is not known, but the problem becomes manageable in D=2, when the corner becomes a point, and the Diff part of the algebra is gone.
- This places us in the realm of 2D gravity.
- This approach carries physical significance because 2D gravity, beyond its intrinsic interest, also describes the spherically symmetric sector of 4D gravity.



* L. Ciambelli, JKG, and L. Varrin, arXiv:2406.07101 [hep-th]

QCS = central extension of UCS_2 *

- In quantum theory, we are interested in projective representations of symmetries. The Bargmann–Mackey theorem states that projective representations are equivalent to ordinary unitary representations of the maximal central extension of the symmetry group. Therefore, to obtain the physical representations, we must consider the central extensions of the group.
- Interestingly, ECS_2 also admits a central extension: the emergence of the central element C turns the translational algebra $\mathbb{R}^2$ into the Heisenberg algebra $\mathbb{H}_2$ .This gives rise to the quantum corner algebra QCS_2, whose irreducible unitary representations we shall investigate.

$$\mathfrak{qcs}_2 = \mathfrak{sl}(2,\mathbb{R}) \ltimes \mathbb{H}_2$$

* The representation theory of QCS_2 are discussed in 2409.10624 and 2507.10683 [hep-th]

Algebra of QCS_2

- The algebra of QCS_2

$$[L_0, L_\pm] = \pm L_\pm, \qquad [L_-, L_+] = 2L_0,$$
$$[L_0, P_\pm] = \pm\frac{1}{2}\,P_\pm\,, \quad [L_\pm, P_\pm] = 0\,, \quad [L_\pm, P_\mp] = \mp P_\pm$$
$$[P_-, P_+] = C$$

- The algebra has two Casimirs, the central element C, and a cubic Casimir

$$\mathscr{C}_{\rm QCS} = C\left(L_0(L_0 + \frac{3}{2}) - L_-L_+ + \frac{1}{16}\right) + \frac{1}{2}\left(L_-P_+^2 + L_+P_-^2 - 2L_0P_-P_+\right)$$

General representations

- The general representation is labeled by $s \in \mathbb{R}^+$; n,k $\in \mathbb{N}$

$$C|n,k\rangle = c|n,k\rangle$$

$$P_-|n,k\rangle = \sqrt{ck}|n,k-1\rangle$$

$$P_+|n,k\rangle = \sqrt{c(k+1)}|n,k+1\rangle$$

$$L_0|n,k\rangle = \left(s+\frac{k}{2}+n+\frac{3}{4}\right)|n,k\rangle$$

$$L_-|n,k\rangle = \sqrt{n(n+2s)}|n-1,k\rangle + \frac{1}{2}\sqrt{k(k-1)}|n,k-2\rangle$$

$$L_+|n,k\rangle = \sqrt{(n+1)(2s+n+1)}|n+1,k\rangle + \frac{1}{2}\sqrt{(k+1)(k+2)}|n,k+2\rangle$$

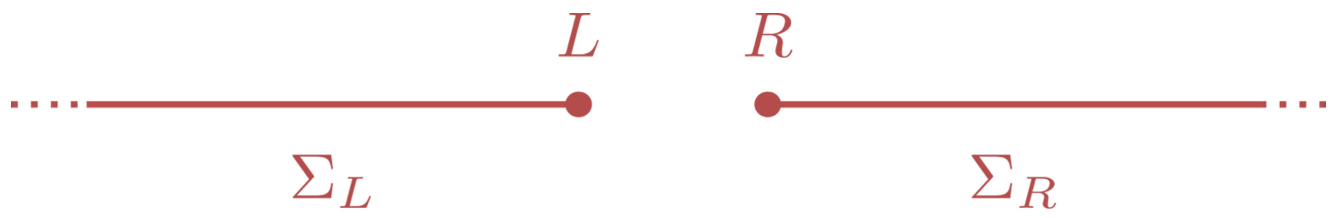
$$\mathscr{C}_{\mathrm{QCS}}\,|n,k\rangle = cs^2\,|n,k\rangle$$

Comments

- The QCS algebra organizes corner charges at the quantum level. In gravity, these charges arise as combinations of metric components at, and near, the corner. The associated Hilbert space can thus be viewed as the quantum counterpart of the geometric data localized at a corner.
- The QCS operators act on this Hilbert space by creating or annihilating "quantum bits of geometry." The vacuum state (the lowest weight state) seems to represent the absence of geometry rather than a flat background.
- The corner algebra thereby becomes an organizing principle for quantum operators. The metric (or connection) is not itself a fundamental quantum datum; only the charge algebra and its representation theory play that role.
- Ultimately, the theory must recover the metric in an appropriate semiclassical limit.

Gluing segments – an idea

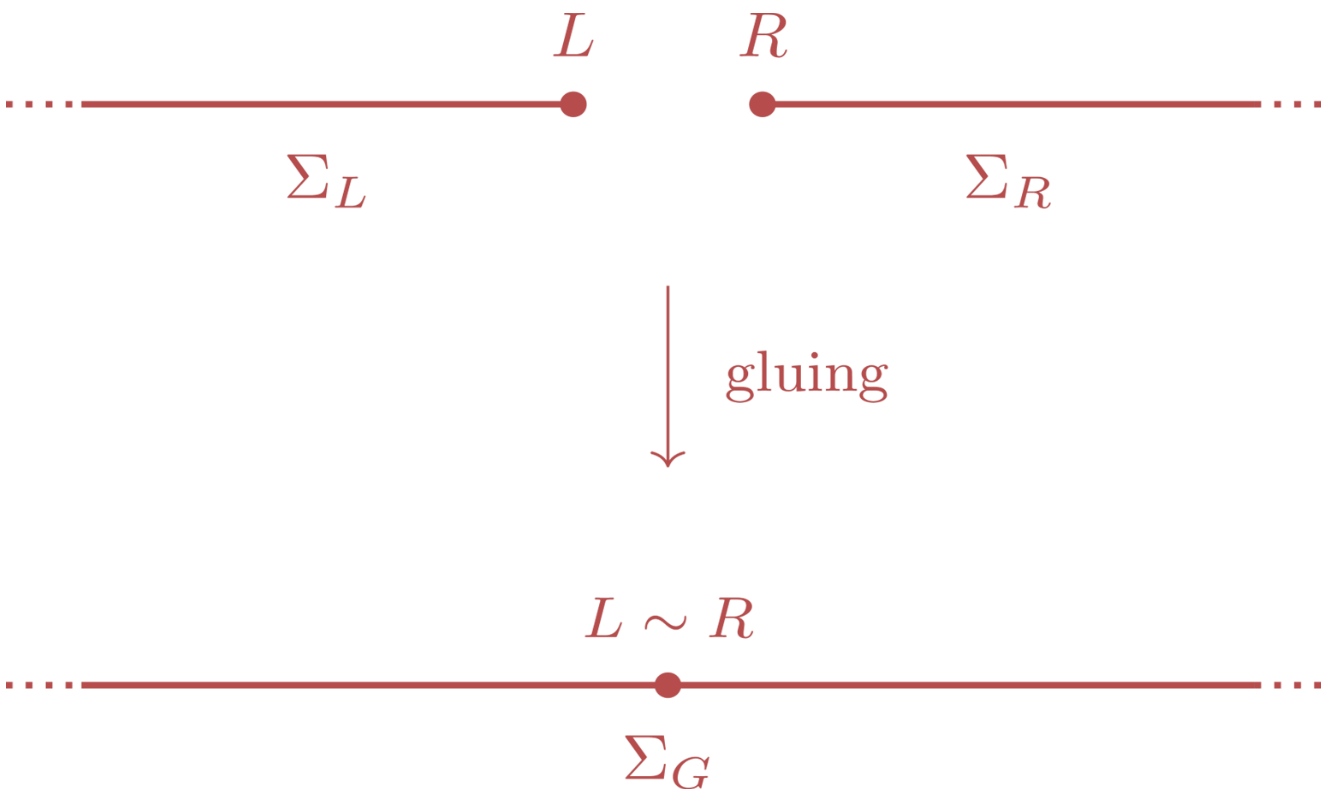
- Having two segments we can glue them into one.



Gluing segments – an idea

- Two segments could be glued into one only when the charges of the L and R corners are equal. The entangled product

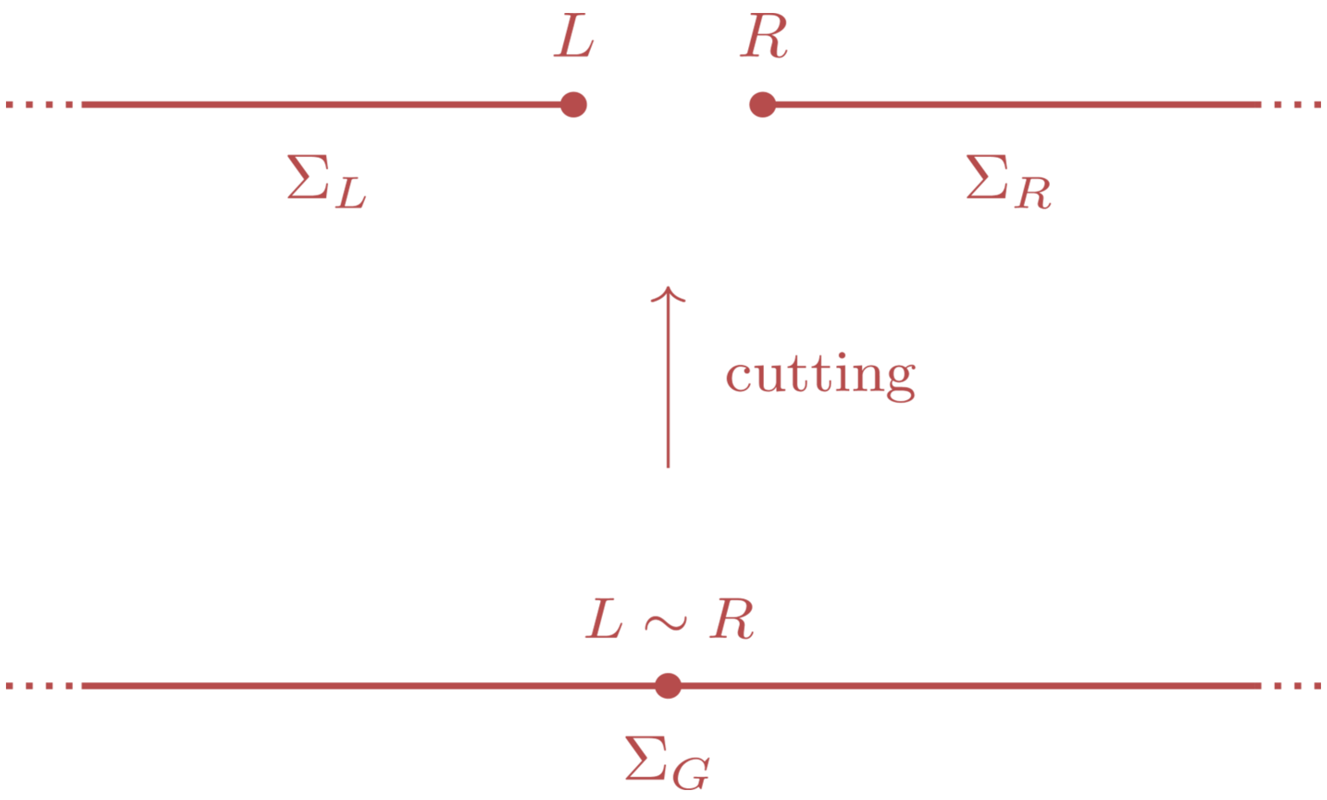
$$\mathscr{H}_G = \mathscr{H}_L \sqcup_{\mathrm{QCS}} \mathscr{H}_R \subset \mathscr{H}_L \otimes \mathscr{H}_R = \tilde{\mathscr{H}}_G,$$



Cutting line into segments

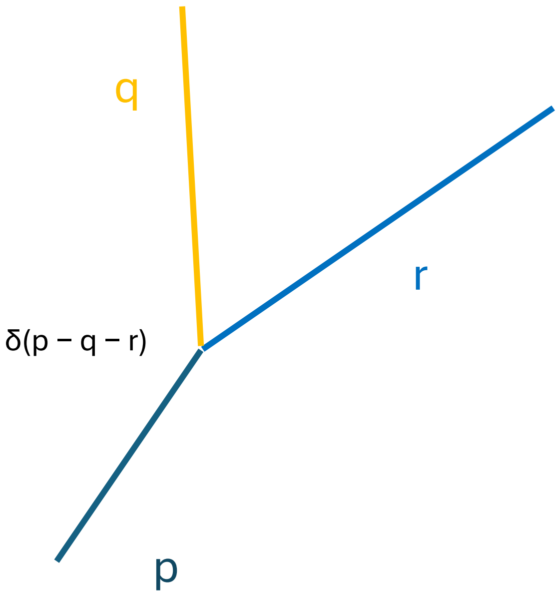
$$\mathscr{H}_G = \mathscr{H}_L \sqcup_{\mathrm{QCS}} \mathscr{H}_R \subset \mathscr{H}_L \otimes \mathscr{H}_R = \tilde{\mathscr{H}}_G,$$

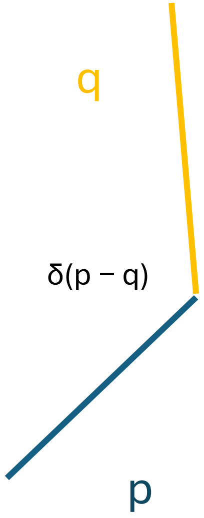
- Knowing how to glue, we can also cut back.



Quantum gluing

- Consider a three-valent vertex in a Feynman diagram. Momentum conservation at the vertex is ensured by the inclusion of the delta function $\delta(p - q - r)$.
- Now, let us turn to a bi-valent vertex. The delta function associated with this vertex is $\delta(p-q)$, reflecting the equality of the incoming and outgoing momenta.
- The momenta in this context are the eigenvalues of momentum operators, i.e. to the Noether charges corresponding to translational symmetry at the corners of the lines.
- We impose momentum conservation at the vertex because the momentum operators constitute its maximal commuting subalgebra.
- To summarize, in the case of a bi-valent vertex in Feynman diagram construction, we impose the equality of the eigenvalues of the maximally commuting set of operators within the Poincare algebra.
- Our approach in the context of corner symmetry reflects this same physical intuition, adapted to the relevant framework.





Quantum gluing

- To perform the gluing, we need to identify the maximal commuting sub-algebra of the QCS, which is 3 dimensional. We then demand that after gluing the eigenvalues of the left and right commuting operators coincide.
- It is convenient to choose as the three commuting operators the triple (C,H,P) with

$$H = L_0 - \frac{1}{2}(L_+ + L_-)$$

$$P = \frac{i}{\sqrt{2c}}(P_+ - P_-)$$

$$H\,|E,p\rangle = E\,|E,p\rangle$$

$$P\,|E,p\rangle = p\,|E,p\rangle$$

$$C\,|E,p\rangle = c\,|E,p\rangle$$

$$\mathscr{C}_{\mathrm{QCS}}\,|E,p\rangle = c\left(s^2 - \frac{1}{16}\right)|E,p\rangle$$

Quantum gluing and splitting

- To describe the gluing, we start from the two segments Hilbert space

$$\tilde{\mathscr{H}}_G = |E, p; c\rangle_L \otimes |E', p'; c'\rangle_R$$

- Then we impose the “Noether charge conservation condition” to obtain

$$\mathscr{H}_G = |E; c\rangle_G = |E, p; c\rangle_L \otimes |E, p; c\rangle_R$$

- The split of a segment into two follows the exact opposite path. Choose the corner point at which split the system. There is now one copy of the Hilbert space associated with the chosen point. The system is doubled taking the diagonal tensor product and then relaxing the gluing conditions, we obtain

$$|E, p; c\rangle \xrightarrow{\text{double}} |E, p; c\rangle_L \otimes |E, p; c\rangle_R \xrightarrow{\text{relax}} |E, p; c\rangle_L \otimes |E', p'; c'\rangle_R$$

- A general state in the Hilbert space associated with the glued/splitted point has the form

$$|\psi, \phi\rangle = \int dp dE\, \psi(E)\phi(p)\, |E, p\rangle_L \otimes |E, p\rangle_R ,$$

Entanglement entropy

- The gluing makes the left and right Hilbert spaces entangled. The density matrix

$$\rho = |\psi, \phi\rangle \otimes \langle \psi, \phi|$$
$$= \left(\int dp dE\, \psi(E)\phi(p)\, |E,p\rangle_L \otimes |E,p\rangle_R \right) \otimes \overline{\left(\int dp' dE'\, \psi(E')\phi(p')\, |E',p'\rangle_L \otimes |E',p'\rangle_R \right)}$$

- The reduced density matrix

$$\rho_{red} = \mathrm{Tr}_L \rho = \int dp dE\, |\psi(E)|^2\, |\phi(p)|^2\, |E,p\rangle \otimes \langle E,p|$$

- And the entanglement entropy

$$S_{ent} = -\mathrm{Tr}\, \rho_{red} \ln \rho_{red}$$
$$= -2 \int_0^\infty \mathrm{d}E\, |\psi(E)|^2 \ln(|\psi(E)|) - 2 \int_{-\infty}^\infty \mathrm{d}p\, |\phi(p)|^2 \ln(|\phi(p)|)$$

L. Ciambelli, JKG, L. Varrin, Arxiv, 2507.16800 [hep-th]

Coherent state entanglement entropy

- We compute the entanglement entropy in a coherent state, i.e., the state that is "as close to the classical configuration as possible".
- A general coherent state of QCA is defined as

$$|\zeta,\alpha\rangle \equiv e^{\nu_\zeta L_+ - \bar{\nu}_\zeta L_-} e^{\frac{1}{\sqrt{c}}(\alpha P_+ - \bar{\alpha} P_-)} |n=0, k=0\rangle$$

- There is a special class of coherent states, called classical states, for which the parameter ν_ζ is a given function of the representation label s. In such states the entanglement entropy scales as

$$S^{\psi_{cl}} \sim s + \frac{1}{2}\ln(s), \quad s \gg 1$$

Spherically symmetric configurations

- Every spherically symmetric metric can be brought to the form

$$ds^2 = g_{ab}dx^a dx^b + \rho^2(x^a)d^2\Omega$$

- The effective two-dimensional action becomes

$$S_{\mathrm{EH}} = \int_{\tilde{M}_2} \mathrm{d}^2x\sqrt{-\hat{g}}\left(\Phi\hat{R}_2 + \Phi^{-\frac{1}{2}}\frac{2}{L^2}\right) - \frac{3}{2}\int_{\partial\tilde{M}_2} \mathrm{d}\Sigma_a \sqrt{-\hat{g}}\hat{g}^{ab}\partial_b\Phi$$

$$\Phi = \frac{\rho^2}{4G\hbar}$$

$$\hat{g}_{ab} = \frac{\rho\, L^2}{(4G\hbar)^{3/2}}\, g_{ab}$$

Charges of spherically symmetric configurations

- There is a one-to-one association of classical charges and generators of the algebra. For example,

$$N_a^b = \frac{\epsilon^{cb}}{\sqrt{-g^{(0)}}} \frac{\rho_{(0)}^2}{4G} g_{ca}^{(0)}$$

- In the case of Schwarzschild spacetime

$$N_1^0 \, N_0^1 \Big|_{hor} \sim \text{Area}^2$$

In Planck units

- On the other hand

$$N_1^0 \leftrightarrow K = \hbar \left(L_0 + \frac{1}{2}(L_+ + L_-) \right)$$

$$N_0^1 \leftrightarrow H = \hbar \left(L_0 - \frac{1}{2}(L_+ + L_-) \right)$$

Area operator

- Therefore, we identify the product of operators K H with Area^2 operator. (Alternatively, following Donnelly&Freidel we can use the sl(2,$\mathbb{R}$) Casimir.)
- The expectation value of K H in a state is the expectation value Area^2 in this state. For Schwarzschild we compute

$$\boxed{\langle cl|\,\mathrm{Area}^2\,|cl\rangle} \sim \langle cl|\,K\,H\,|cl\rangle \sim \langle cl|\,K\,|cl\rangle\,\langle cl|\,H\,|cl\rangle + \cdots \sim \boxed{s^2} + \cdots$$

- But in the large s limit the entanglement entropy scales as s, and therefore we find the area law

$$\boxed{S^{cl} \sim \sqrt{\langle cl|\,\mathrm{Area}^2\,|cl\rangle}}$$

Summary & what's next

- We investigated the Quantum Corner Conjecture, using two-dimensional gravity as a toy model.
  - We identified the two-dimensional quantum corner algebra and analyzed its representation theory.
  - We established the gluing and splitting procedures.
  - We computed entanglement entropy of a coherent state. While the connection between these quantities and semiclassical geometry remains to be fully understood, we have derived the area law for appropriate semiclassical states.
- The dynamics of gravity—interpreted as the evolution of the corner—requires further investigation. The Jacobson's gravity-as-thermodynamics paradigm seems to offer a promising conceptual framework.
- Repeat in 3D (cylindrically-symmetric 4D geometries).
- Repeat in 4D (general case).

Dedicated to Rob Leigh