Redshift and position drift in cosmology

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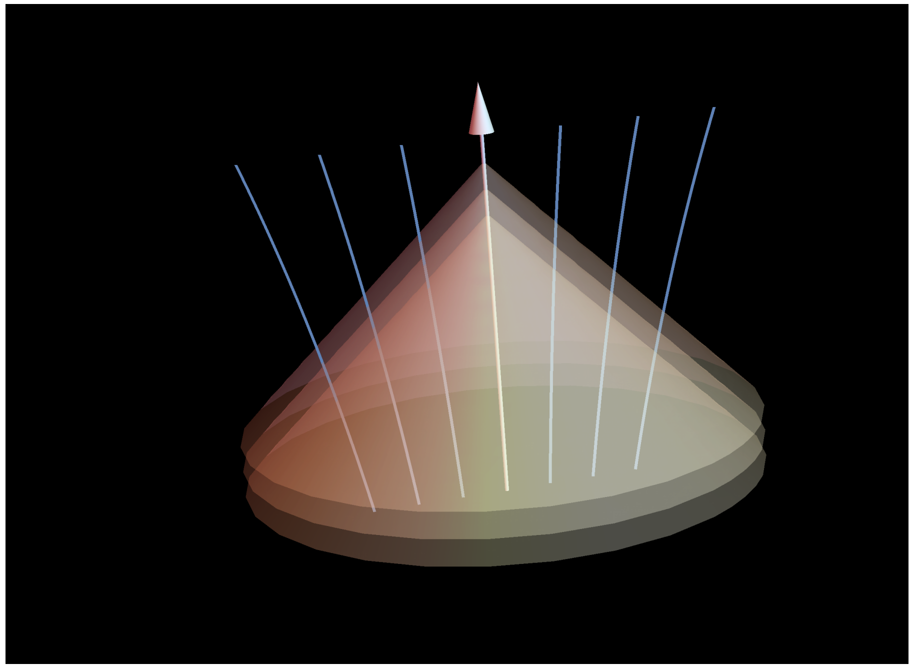
What are the drifts good for?

Drifts = slow variations of apparent position and redshift in observer's time



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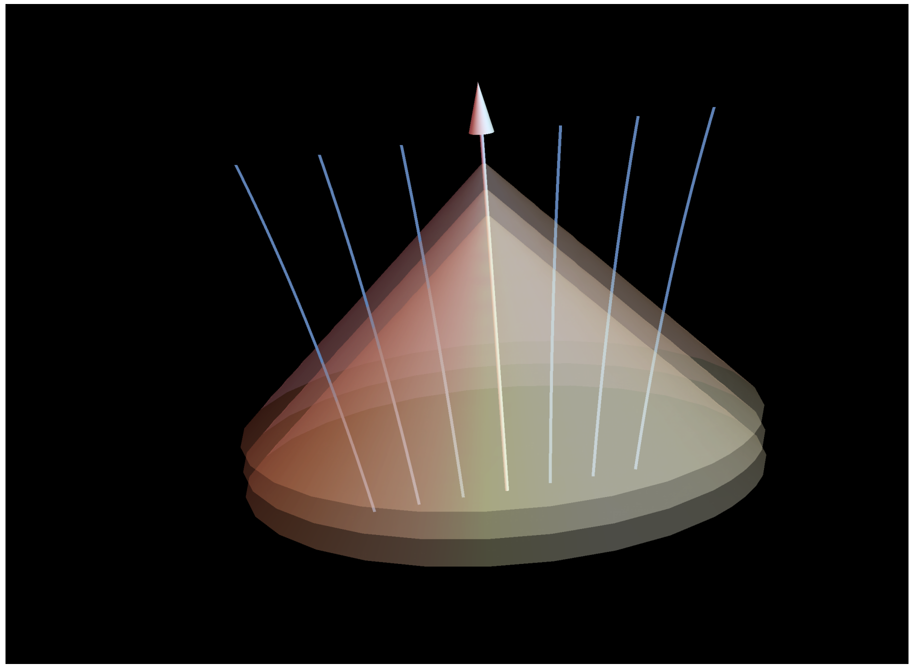

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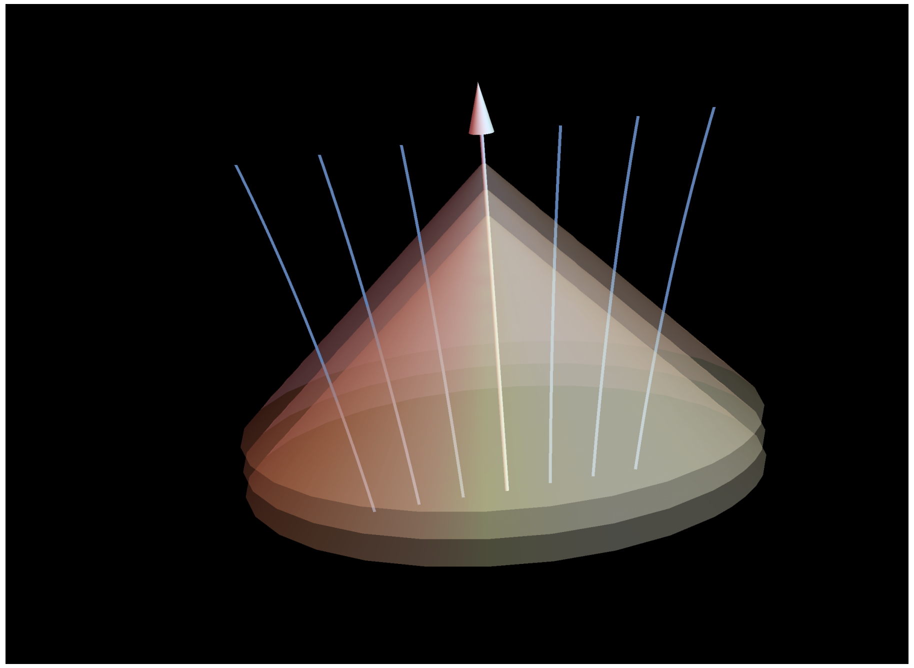

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$$\frac{dz}{dt} = (1+z)\,H_0 - H(z)$$

FLRW Universe, no perturbations, no peculiar motions

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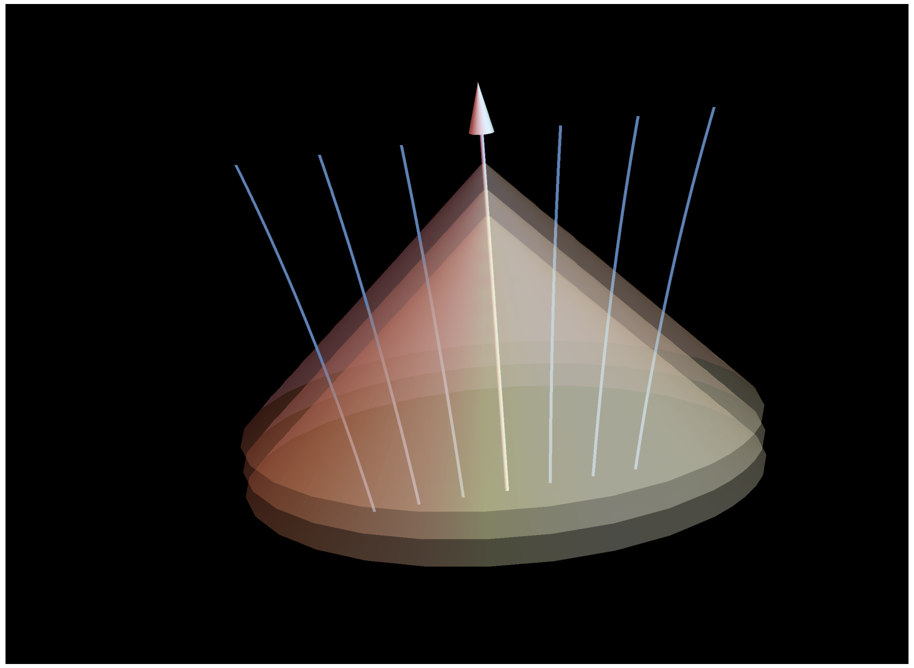

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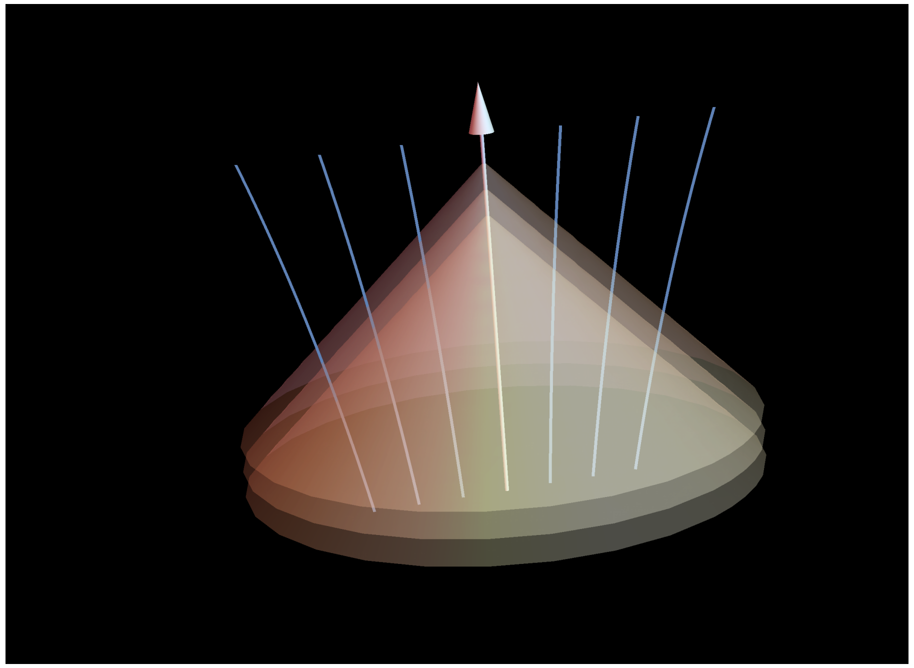

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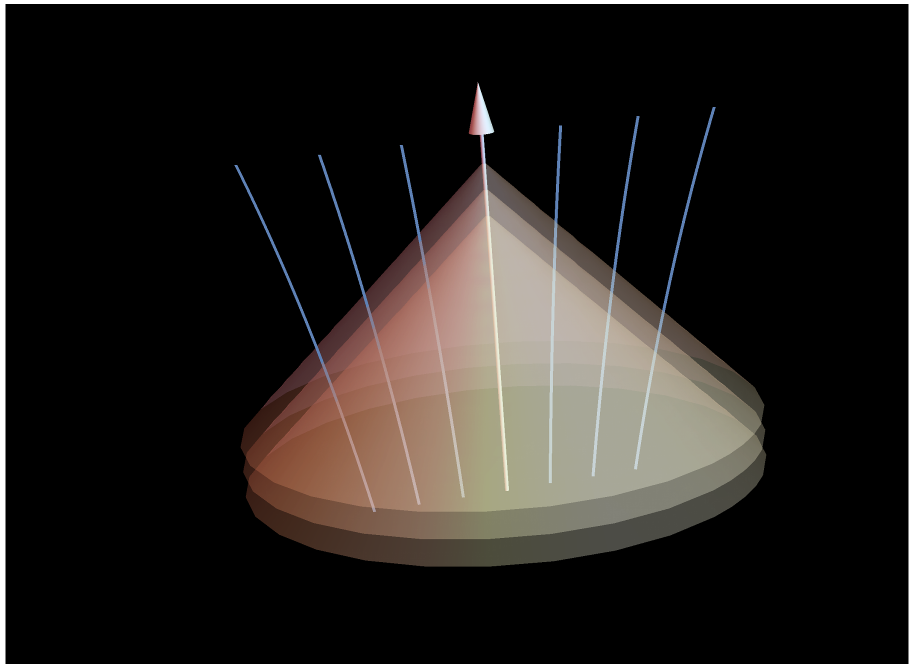

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a lot of theoretical work recently…

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position drift

kinematical effect already measured: aberration drift due to the motion of the Solar System
galactic acceleration $\approx 0.7\,\mathrm{cm}\cdot\mathrm{s}^{-1}\cdot\mathrm{yr}^{-1}$, corresponds to the drift of $\approx 5\,\mu\mathrm{as}\cdot\mathrm{yr}^{-1}$

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redshift drift

no measurements, only prospects: E-ELT + CODEX - Ly-alpha forest variations, SKA (HI lines variations)

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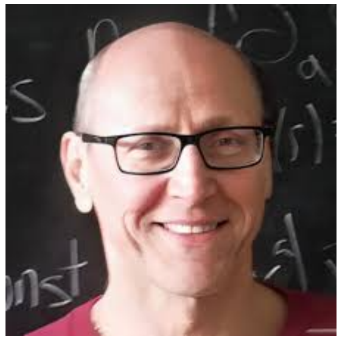
Theoretical work

Bianchi I models [Fleury *et al* 2015…], off-center observers in Lemaître-Tolman-Bondi models [Krasiński-Bolejko 2011…], Newtonian N-body simulations [Koksbang 2021], hydrodynamic relativity simulations [Koksbang *et al* 2024]…

- Quercellini *et al*, *„Real time cosmology"*, Phys. Rep. **521**, 95 (2012).
- O. H. Marcori *et al*, *„Direction and redshift drifts for general observers and their applications in cosmology"*, Phys. Rev. D **98**, 023517 (2018)

My path: from the PhD project to the drifts



PhD project: isolated
and dynamical horizons

null geodesics,
congruences

geometric and
wave optics in GR

**redshift and
position drifts**

cosmology

Introduction

Project in collaboration with Asta Heinesen (Niels Bohr Institute, Copenhagen)

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- Derive the drifts for small redshifts under as few assumptions as possible, using the cosmographic approach
- How can we constrain a general cosmological model using drift measurements?

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Computationally heavy project

Need for computationally assisted tensor algebra: `xAct` Mathematica package [J. Martín-García 2002-2024]

Introduction

Cosmography

Keeping the model as general as possible, no assumptions regarding the matter flow or geometry (except geodesic).

Local approach: expand all relevant quantities in distances or, better, redshifts. Taylor series coefficients = model parameters

Dependence on the position on the sky: use scalar and vector spherical harmonics decomposition

Potentially large, but finite number of parameters to describe locally the geometry of the Universe

[Kristian-Sachs 1965], [Ellis *et al* 1985], [O. Umeh 2013], [Clarkson and Umeh 2011], [Maartens *et al* 2023], [A. Heinesen 2021]

Hubble relation becomes direction-dependent: framework allows for anisotropic expansion, but remains fairly agnostic regarding the geometry

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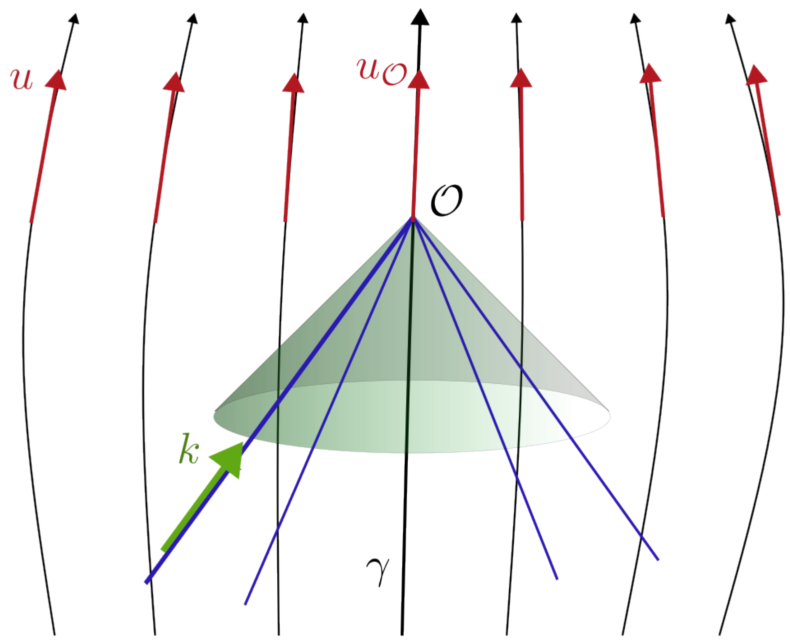
- **Include the drifts as observables**
- **What kind of information about the spacetime geometry can you extract from measurements at fairly small distances?**

Physical model



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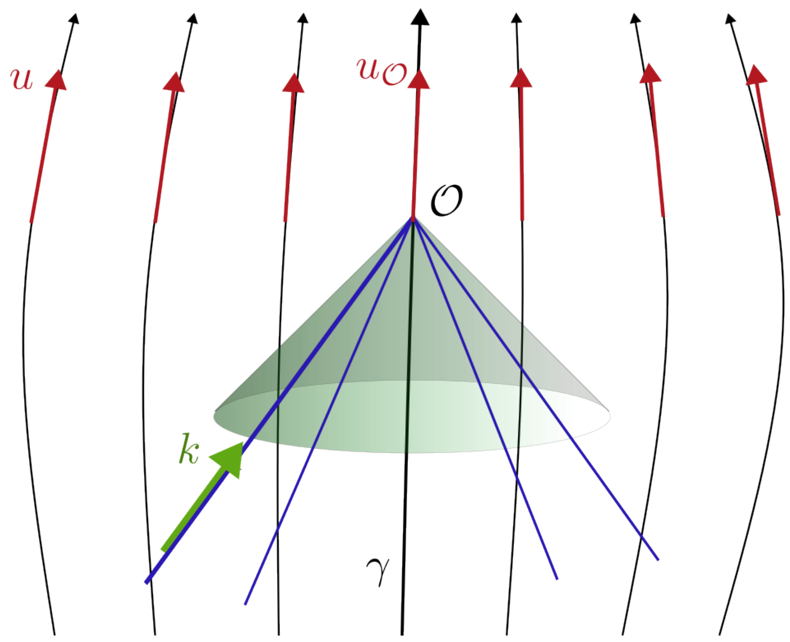

Cosmological time-like congruence of matter u^μ

Geodesic congruence $\nabla_u u^\mu = 0$

Observation point $\mathcal{O}$

Observer $u^\mu_{\mathcal{O}}$ and all light sources belong tho the congruence

neglecting peculiar motions and peculiar accelerations

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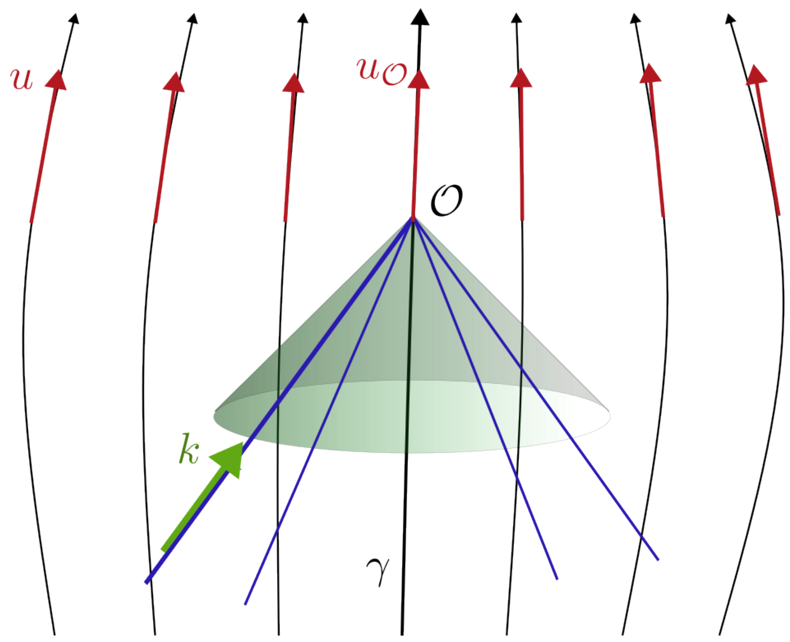

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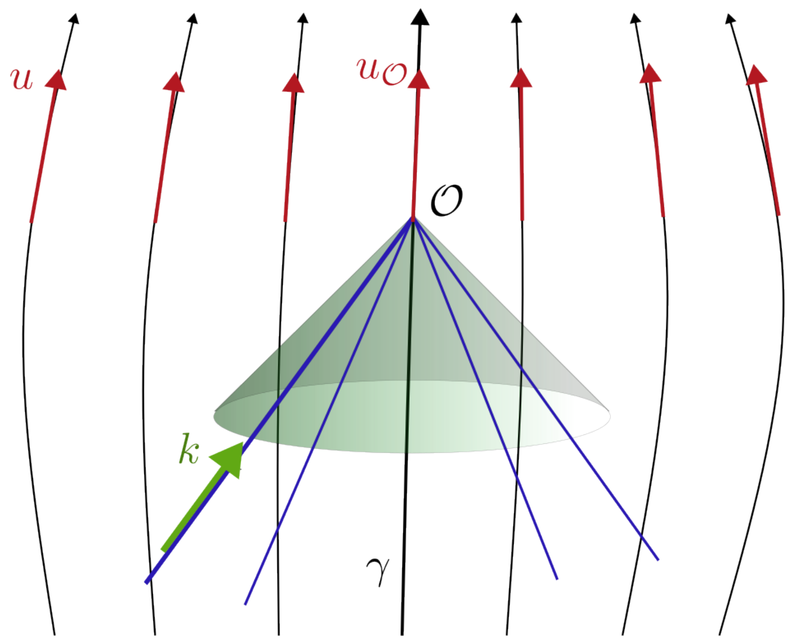

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$$\nabla_\nu u_\mu = \frac{1}{3}\theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}$$

non-isotropic matter motion

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3}R_{\mu\alpha\nu\beta}\Big|_{\mathscr{O}} x^\alpha x^\beta + O(x^3)$$

arbitrary geometry in Riemann's normal coordinates

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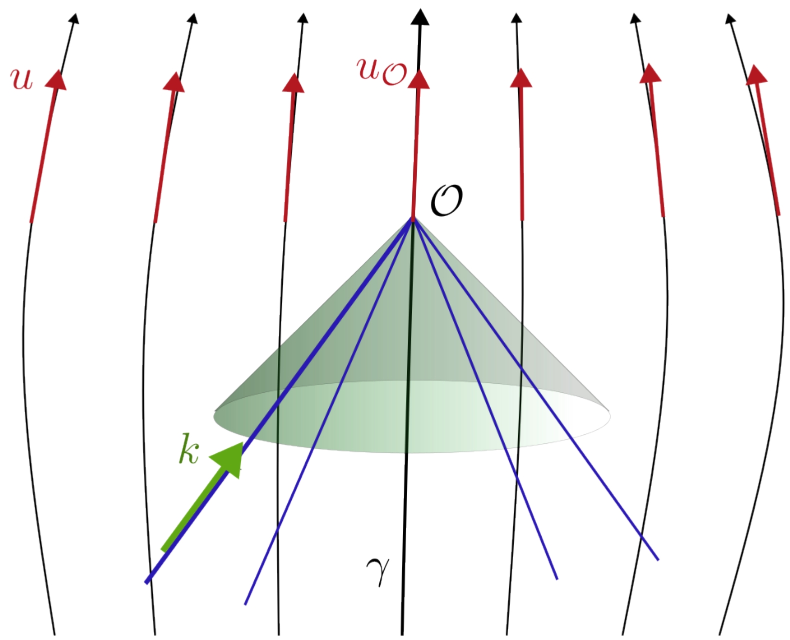

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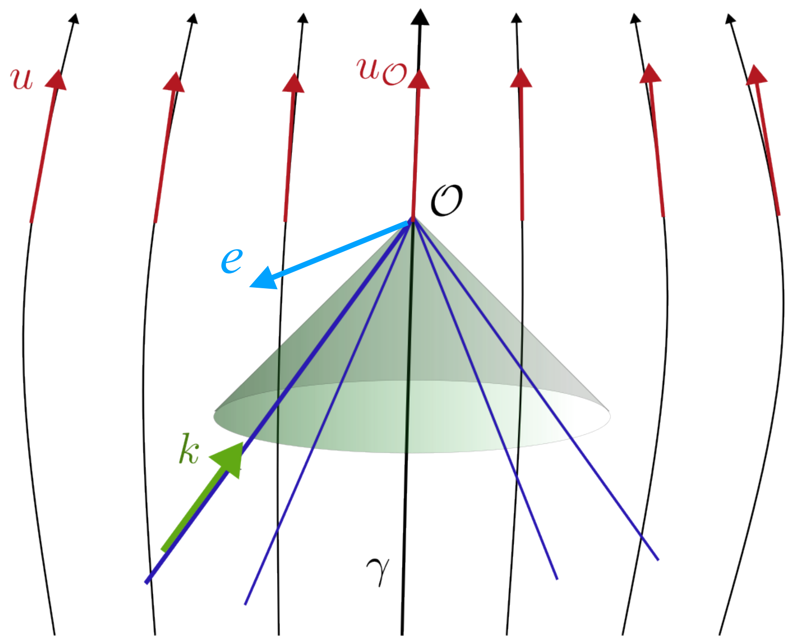
Only a finite number of parameters needed to describe (locally) every model

Decomposition of the drifts



Redshift drift

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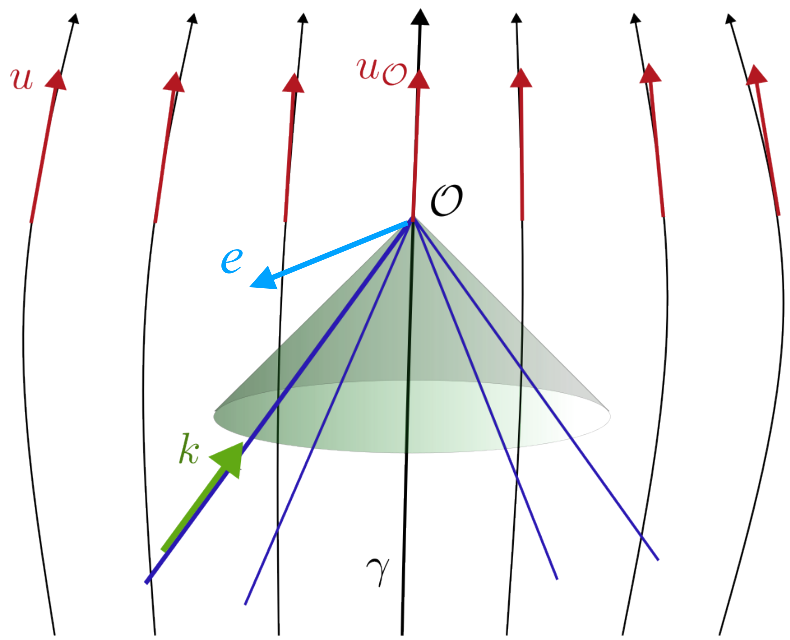

Redshift drift

Taylor expansion in the affine parameter along the light ray

$$\frac{dz}{d\tau_{\mathcal{O}}} \equiv \xi(\lambda, e^i) = {}^{(1)}\xi(e^i)\,\lambda + {}^{(2)}\xi(e^i)\,\lambda^2 + O(\lambda^3)$$

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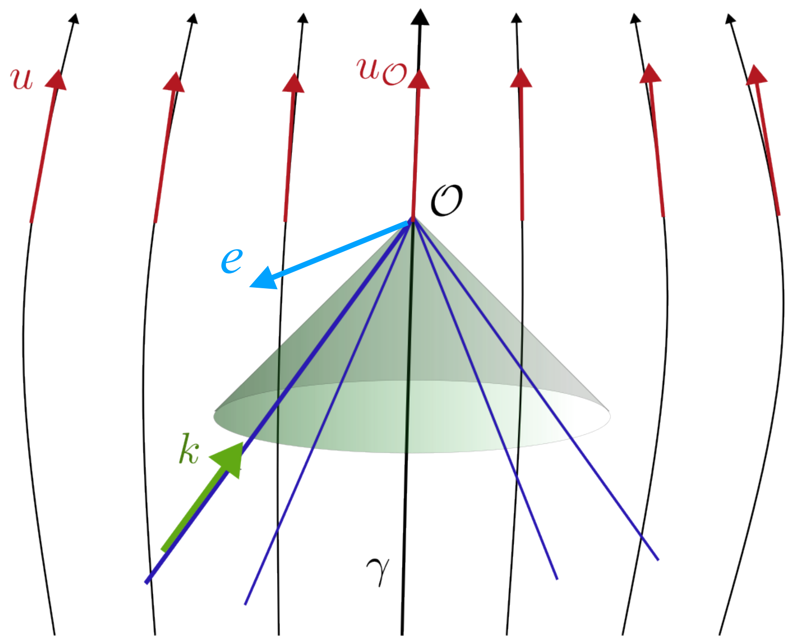

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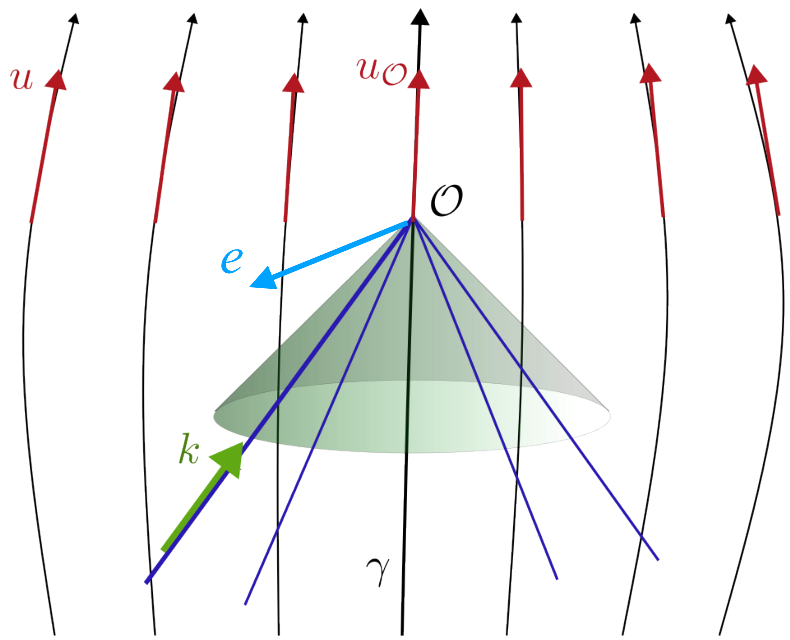
Multipole expansion of the coefficients on the celestial sphere

$${}^{(k)}\xi(e^i) = {}^{(k)}\xi_0 + {}^{(k)}\xi_i\, e^i + {}^{(k)}\xi_{\langle ij\rangle}\, e^i\, e^j + {}^{(k)}\xi_{\langle ijk\rangle}\, e^i\, e^j\, e^k + \cdots$$



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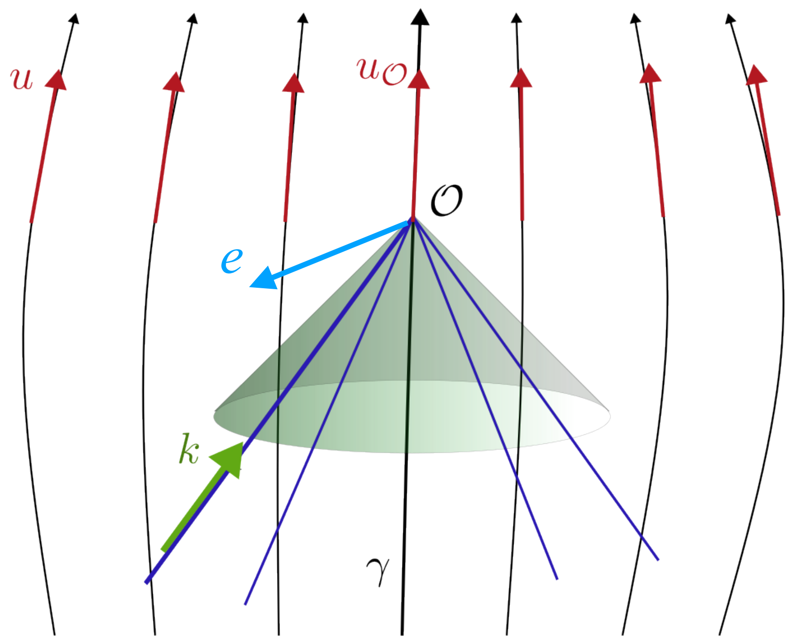

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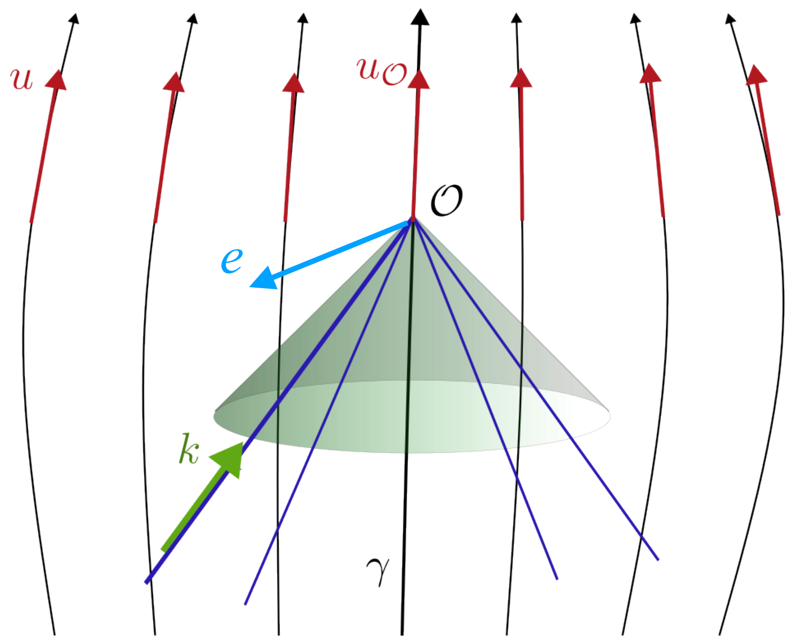

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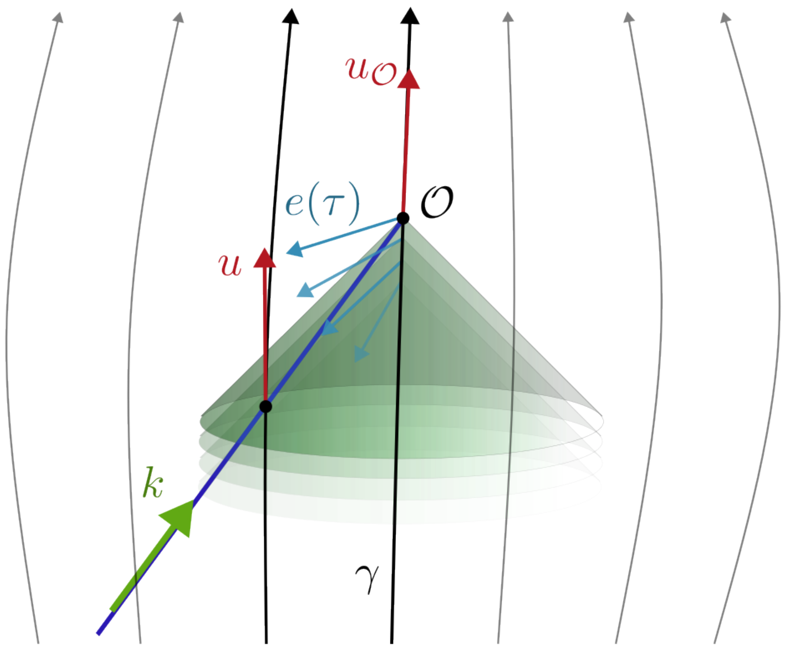
In terms of the redshift (observable)

$$\xi(z, e^i) = {}^{(1)}\hat{\xi}(e^i)\, z + {}^{(2)}\hat{\xi}(e^i)\, z^2 + O(z^3)$$

$$ {}^{(1)}\hat{\xi}(e^i) = -\frac{{}^{(1)}\xi(e^i)}{\mathfrak{H}_{\mathcal{O}}(e^i)}$$

Ratio of functions given by a finite number of multipoles, again no truncation!

Decomposition of the drifts



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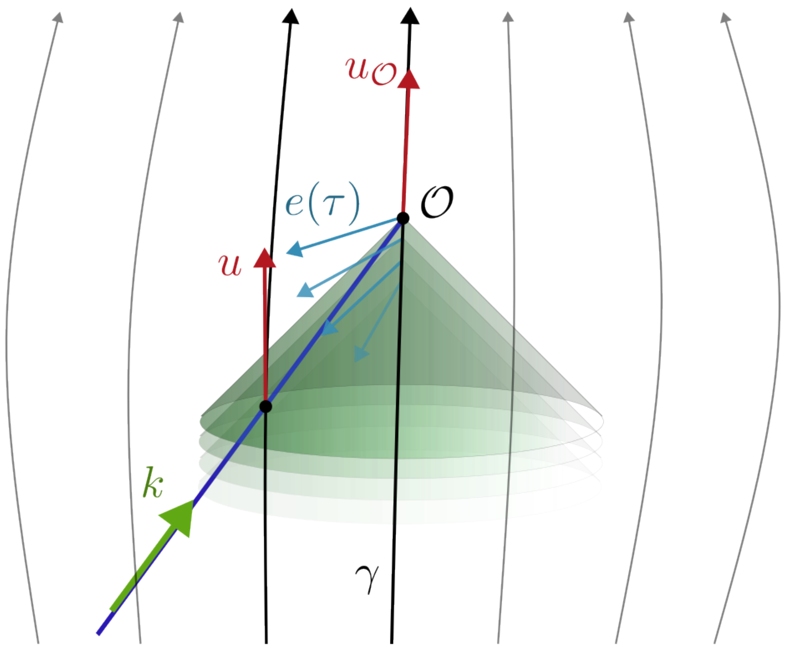

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$$\mathscr{D}_{u_\mathcal{O}} e^\mu = \nabla_{u_\mathcal{O}} e^\mu + \left(a^\mu_\mathcal{O} u_{\mathcal{O}\,\nu} - u^\mu_\mathcal{O} a_{\mathcal{O}\,\nu} \right) e^\nu$$

Fermi-Walker derivative

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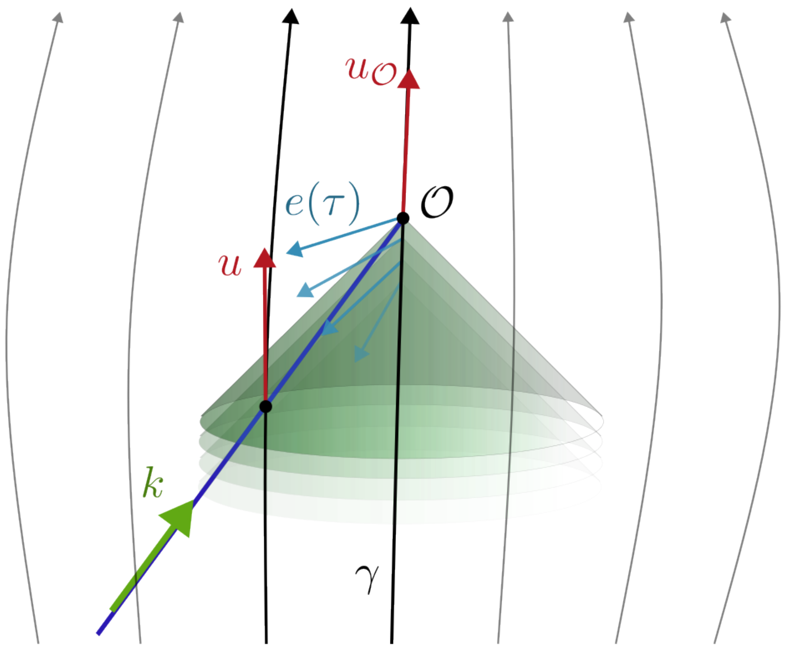

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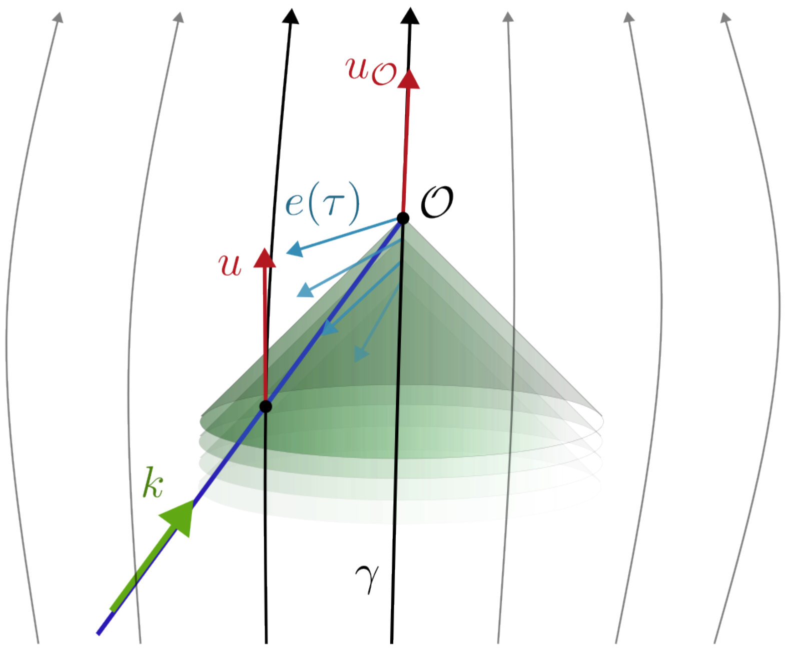

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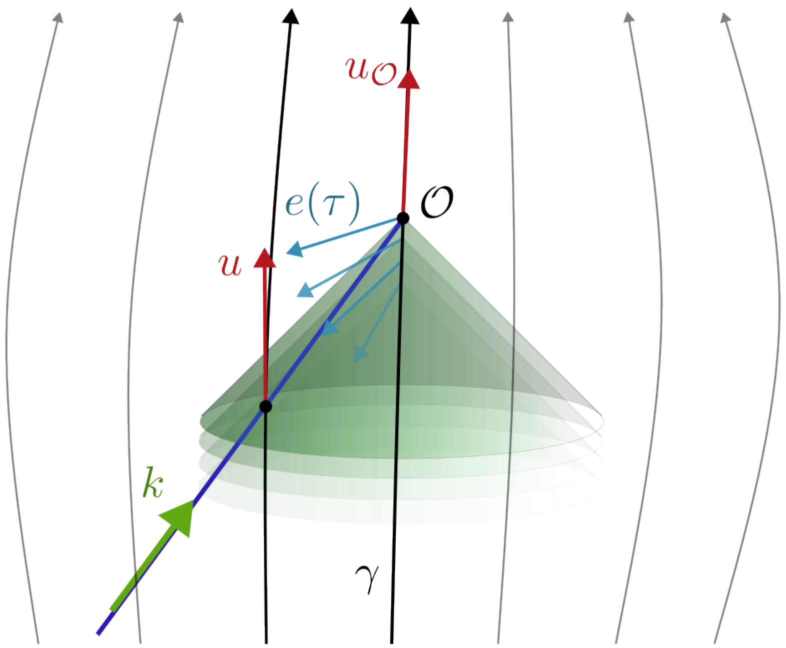
Projection to the transverse subspace

Decomposition of the drifts



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Can be related to the decomposition into vector spherical harmonics on the celestial sphere (toroidal, poloidal)

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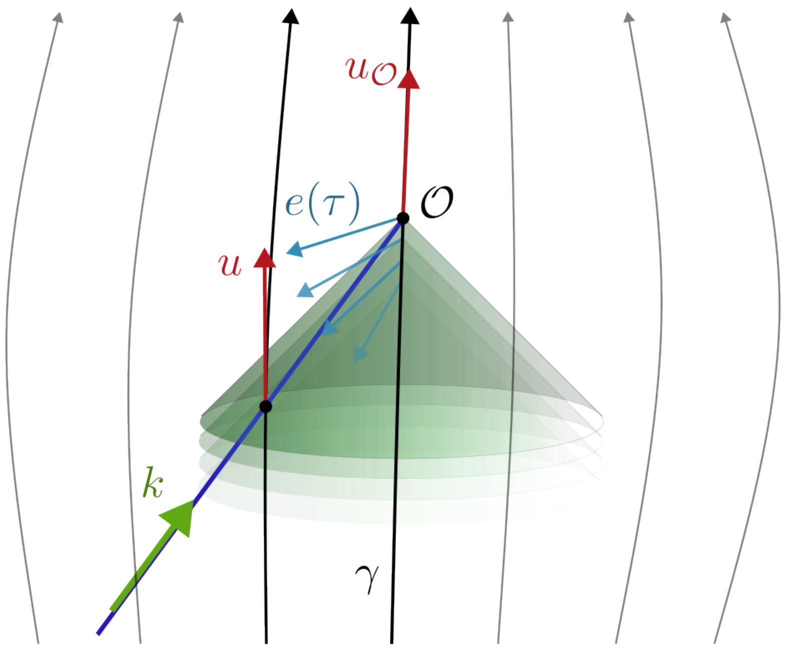
The multipole series terminates at a finite order

Decomposition of the drifts



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$${}^{(0)}\kappa^\sigma = p^\sigma{}_\mu e^\alpha \, {}^{(0)}\kappa^\mu{}_\alpha \qquad {}^{(1)}\kappa^\sigma = p^\sigma{}_\mu \left[{}^{(1)}\kappa^\mu_0 + e^\alpha \, {}^{(1)}\kappa^\mu{}_\alpha + e^\alpha e^\beta \, {}^{(1)}\kappa^\mu{}_{\alpha\beta} + e^\alpha e^\beta e^\gamma \, {}^{(1)}\kappa^\mu{}_{\alpha\beta\gamma} \right]$$

$$p_{\mu\nu} \equiv p_{\mu\nu}(e^\alpha) = h_{\mu\nu} - e_\mu e_\nu = g_{\mu\nu} + u_\mu u_\nu - e_\mu e_\nu$$ Projection to the transverse subspace

The multipole series terminates at a finite order

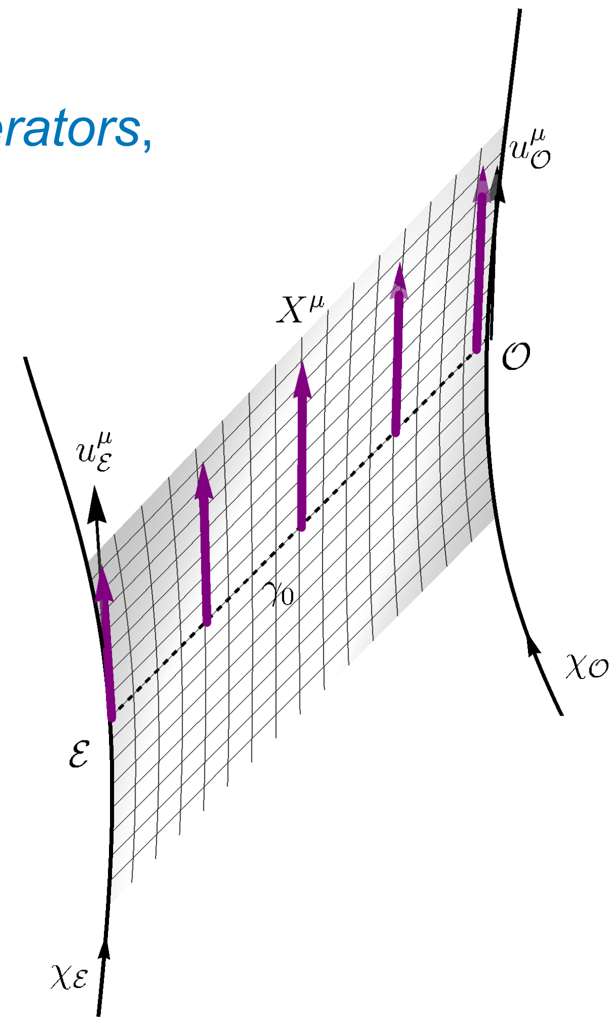
In terms of the redshift (observable)

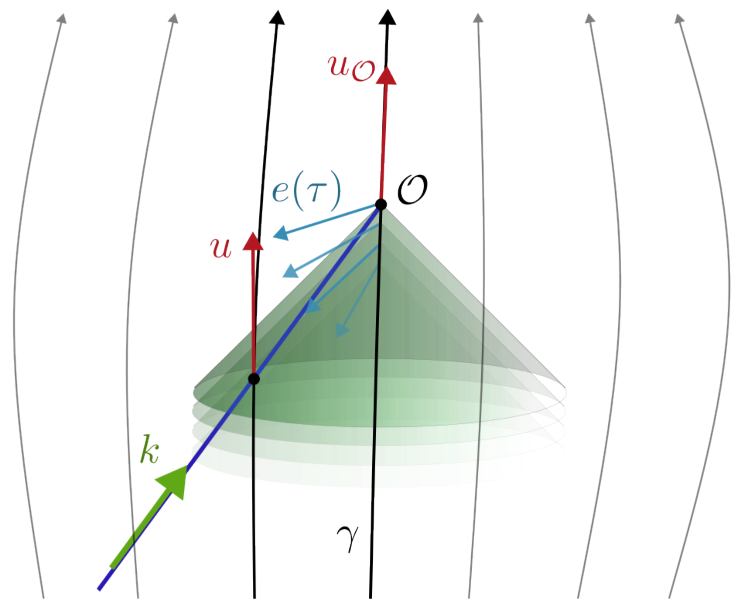
$$\kappa^\sigma_{\mathcal{O}} = {}^{(0)}\kappa^\sigma_{\mathcal{O}} - \left. \frac{{}^{(1)}\kappa^\sigma}{\mathfrak{H}} \right|_{\mathcal{O}} z + O(z^2)$$

Again, ratio of functions given by a finite number of multipoles, again no truncation!

How do you compute the drifts?

- MK, J. Kopiński, *Optical drift effects in general relativity,* JCAP **03** (2018) 012
- M. Grasso, MK, J. Serbenta, *Geometric optics in general relativity using bi-local operators*, Phys. Rev. D **99** (2019) 6, 064038





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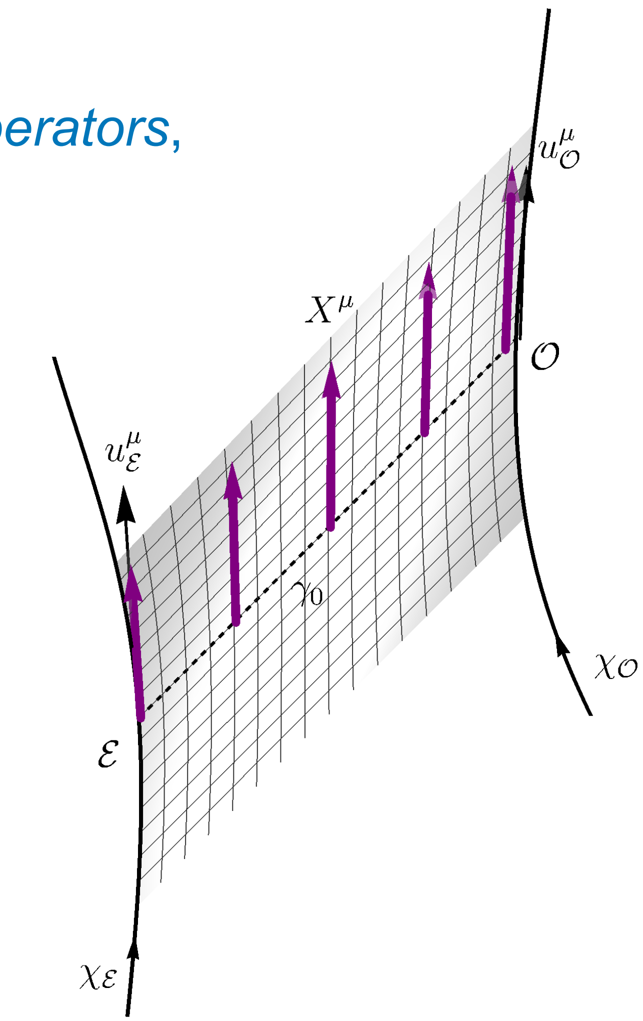


Observer's time flow vector field X^μ

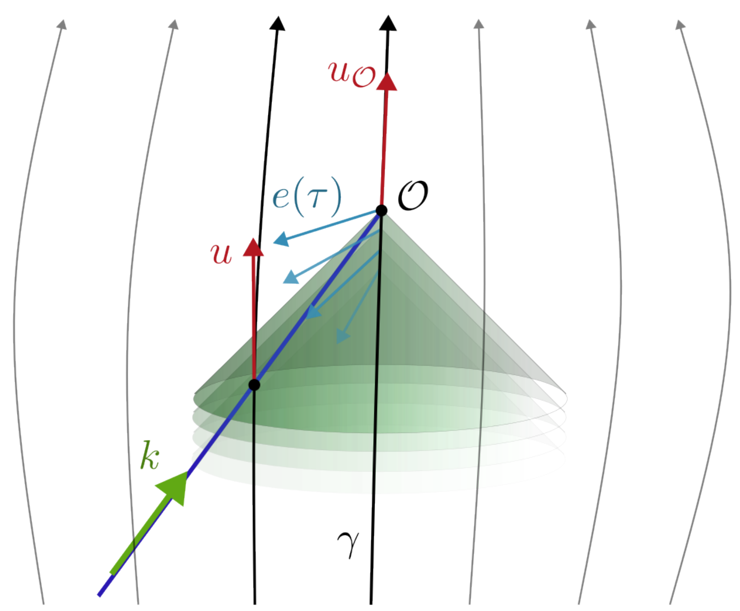
$$\nabla_k \nabla_k X^\mu - R^\mu{}_{\nu\alpha\beta}\, k^\nu\, k^\alpha\, X^\beta = 0$$

Geodesic deviation equation

$$X^\mu \Big|_\mathcal{O} = u^\mu_\mathcal{O}$$

$$X^\mu \Big|_\mathcal{E} = \frac{1}{1+z} u^\mu_\mathcal{E}$$

boundary values



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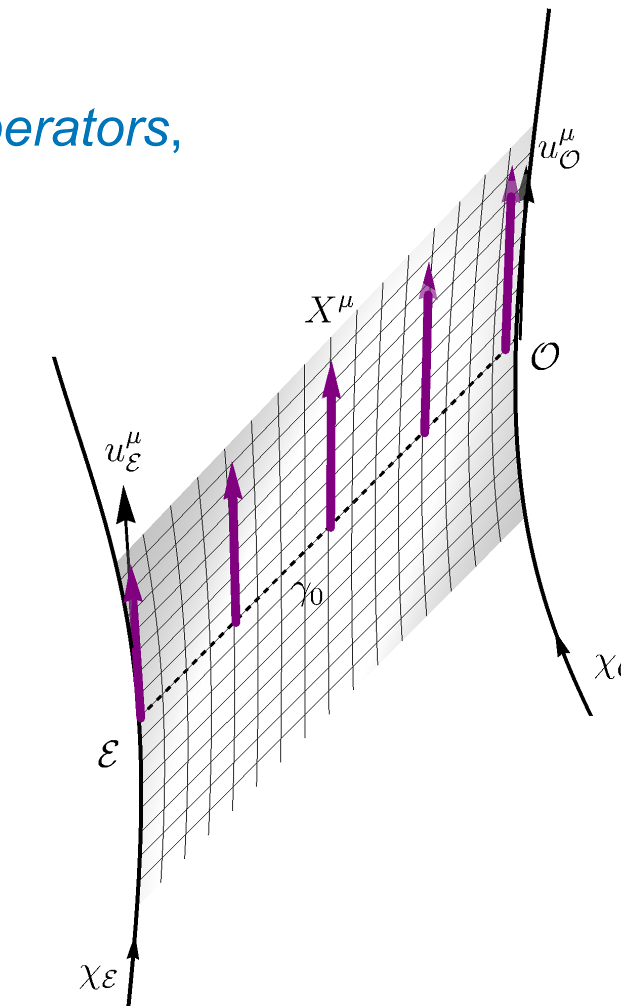

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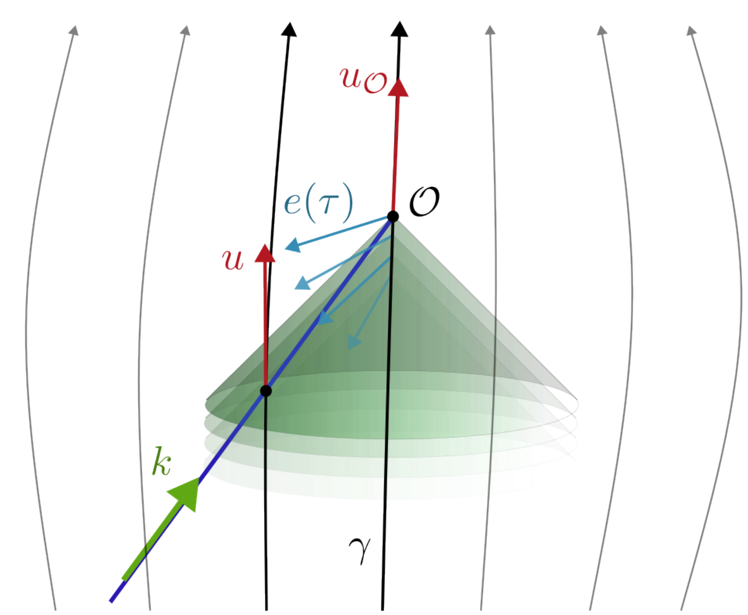
boundary values

$$\kappa^\mu_\mathcal{O} = \mathcal{D}_{u_\mathcal{O}} e^\mu$$

Position drift

$$\mathcal{D}_{u_\mathcal{O}} e^\mu = \nabla_{u_\mathcal{O}} e^\mu + \left(a^\mu_\mathcal{O}\, u_{\mathcal{O}\,\nu} - u^\mu_\mathcal{O}\, a_{\mathcal{O}\,\nu} \right)\, e^\nu$$

Fermi-Walker derivative



$$\nabla_{u_\mathcal{O}} \ln(1+z) = \nabla_X \ln E_\mathcal{E} - \nabla_X \ln E_\mathcal{O}$$

Redshift drift

How do you compute the drifts?

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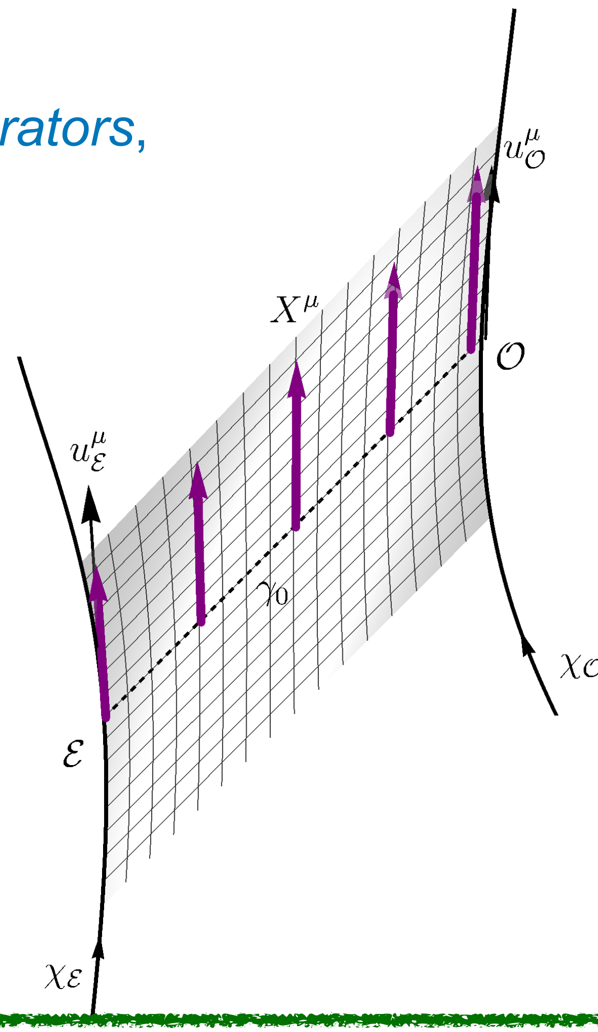


Observer's time flow vector field X^μ

$$\nabla_k \nabla_k X^\mu - R^\mu{}_{\nu\alpha\beta}\, k^\nu\, k^\alpha\, X^\beta = 0$$ Geodesic deviation equation

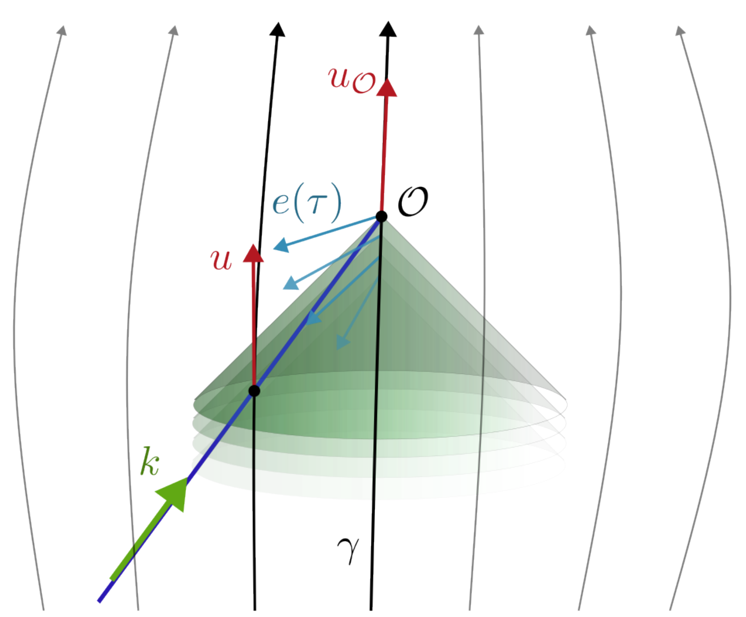
$$X^\mu\Big|_\mathscr{O} = u^\mu_\mathscr{O}$$

$$X^\mu\Big|_\mathscr{E} = \frac{1}{1+z} u^\mu_\mathscr{E}$$ boundary values

$$\kappa^\mu_\mathscr{O} = \mathscr{D}_{u_\mathscr{O}} e^\mu$$ Position drift

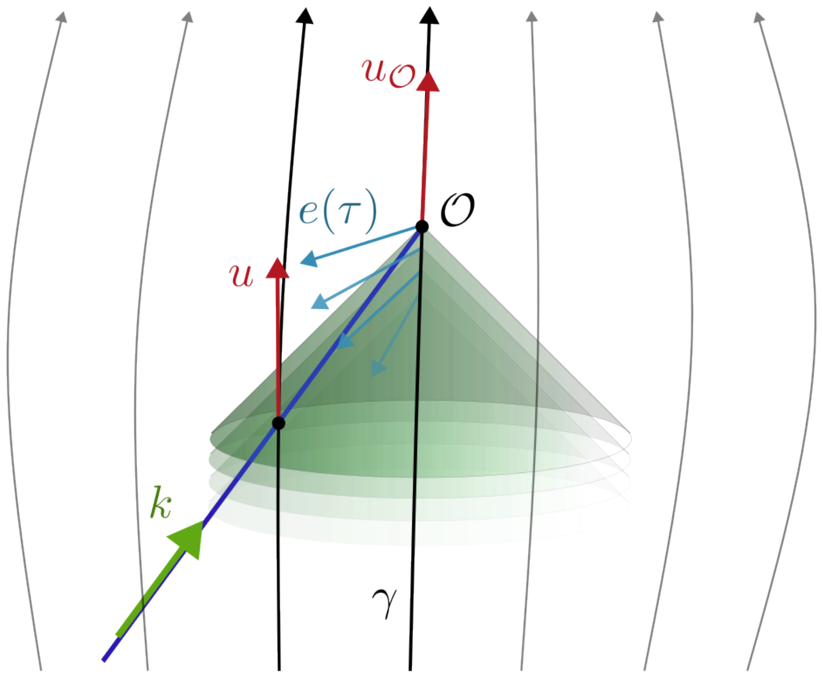
$$\mathscr{D}_{u_\mathscr{O}} e^\mu = \nabla_{u_\mathscr{O}} e^\mu + \left(a^\mu_\mathscr{O}\, u_{\mathscr{O}\,\nu} - u^\mu_\mathscr{O}\, a_{\mathscr{O}\,\nu}\right)\, e^\nu$$ Fermi-Walker derivative

$$\nabla_X e^\mu\Big|_\mathscr{O}$$

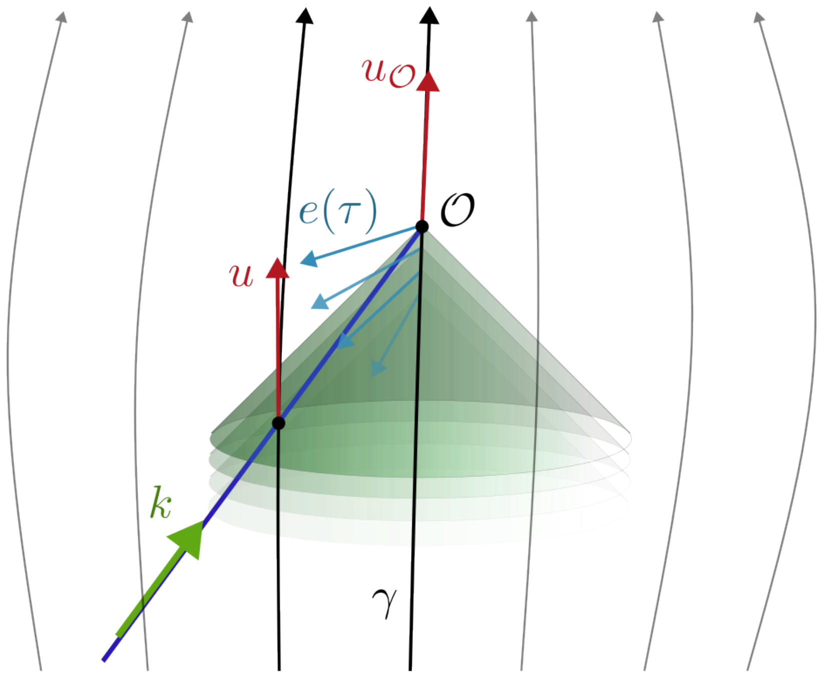


$$\nabla_{u_\mathscr{O}} \ln(1+z) = \nabla_X \ln E_\mathscr{E} - \nabla_X \ln E_\mathscr{O}$$ Redshift drift

Position drift



Position drift



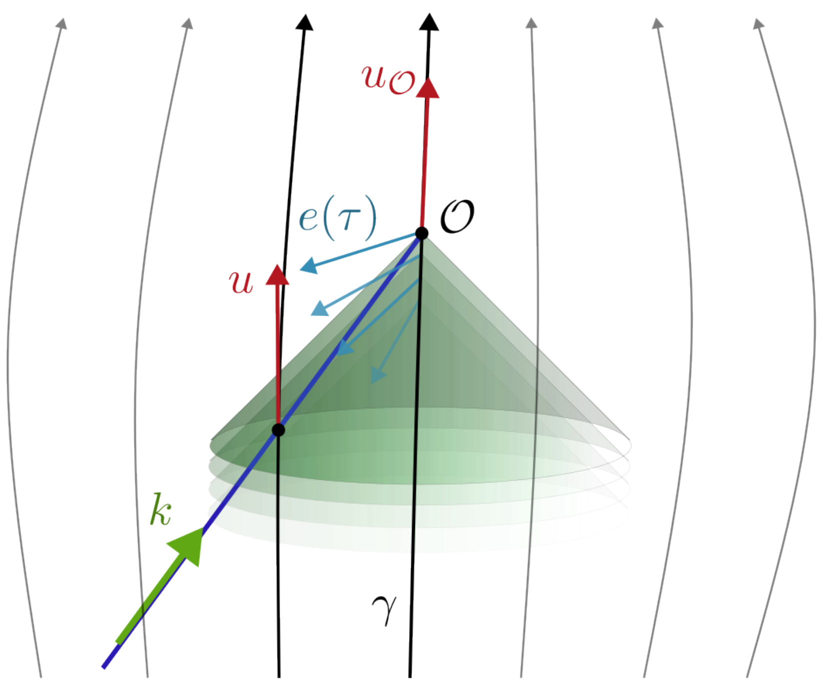
$$\kappa^\sigma|_{\mathscr{O}} = {}^{(0)}\kappa^\sigma|_{\mathscr{O}} - \left.\frac{{}^{(1)}\kappa^\sigma}{\mathfrak{H}}\right|_{\mathscr{O}} z + O(z^2)$$

$${}^{(0)}\kappa^\sigma = p^\sigma{}_\mu\, e^\alpha\, {}^{(0)}\kappa^\mu{}_\alpha$$

$${}^{(1)}\kappa^\sigma = p^\sigma{}_\mu\left[{}^{(1)}\kappa^\mu_0 + e^\alpha\ {}^{(1)}\kappa^\mu{}_\alpha + e^\alpha\, e^\beta\, {}^{(1)}\kappa^\mu{}_{\langle\alpha\beta\rangle} + e^\alpha\, e^\beta\, e^\gamma\, {}^{(1)}\kappa^\mu{}_{\langle\alpha\beta\gamma\rangle}\right]$$

$$\mathfrak{H}_{\mathscr{O}} = \frac{1}{3}\theta_{\mathscr{O}} + \sigma_{\mathscr{O}\,\mu\nu}\, e^\mu_{\mathscr{O}}\, e^\nu_{\mathscr{O}}$$

Position drift



$$\kappa^\sigma|_{\mathcal{O}} = {}^{(0)}\kappa^\sigma|_{\mathcal{O}} - \left.\frac{{}^{(1)}\kappa^\sigma}{\mathfrak{H}}\right|_{\mathcal{O}} z + O(z^2)$$

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$${}^{(1)}\kappa^\sigma = p^\sigma{}_\mu\left[{}^{(1)}\kappa^\mu_0 + e^\alpha\ {}^{(1)}\kappa^\mu{}_\alpha + e^\alpha\, e^\beta\, {}^{(1)}\kappa^\mu{}_{\langle\alpha\beta\rangle} + e^\alpha\, e^\beta\, e^\gamma\, {}^{(1)}\kappa^\mu{}_{\langle\alpha\beta\gamma\rangle}\right]$$

$$\mathfrak{H}_{\mathcal{O}} = \frac{1}{3}\theta_{\mathcal{O}} + \sigma_{\mathcal{O}\,\mu\nu}\, e^\mu_{\mathcal{O}}\, e^\nu_{\mathcal{O}}$$

$${}^{(0)}\kappa^\sigma{}_\mu = \sigma^\sigma{}_\mu + \omega^\sigma{}_\mu$$ ⟵ at $\mathcal{O}$

$${}^{(1)}\kappa^\sigma_0 = \frac{1}{6}h^{\alpha\beta}D_\alpha(\sigma^\mu{}_\beta + \omega^\mu{}_\beta) - \frac{1}{4}R^\mu{}_\nu u^\nu$$

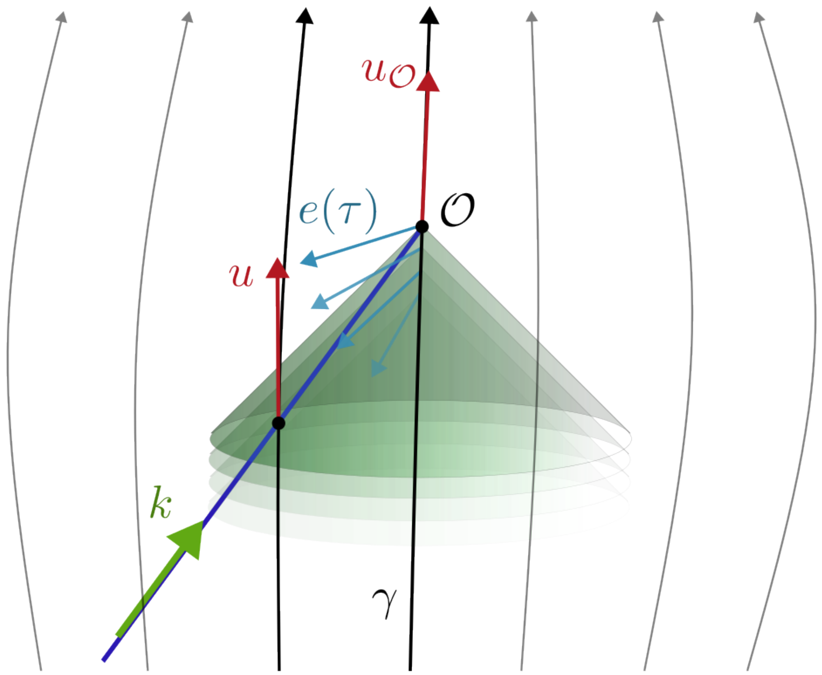
$${}^{(1)}\kappa^\mu{}_\alpha = -\,\theta(\sigma^\mu{}_\alpha + \omega^\mu{}_\alpha) - (\sigma^\beta{}_\alpha + \omega^\beta{}_\alpha)\,(\sigma^\mu{}_\beta + \omega^\mu{}_\beta) + \frac{1}{2}R^\mu{}_\alpha + C^\mu{}_{\nu\alpha\beta}\, u^\nu\, u^\beta - \frac{4}{15}h^{\gamma\beta}\,(\sigma^\mu{}_\gamma + \omega^\mu{}_\gamma)\,\sigma_{\alpha\beta}$$

$${}^{(1)}\kappa^\mu{}_{\alpha\beta} = \frac{1}{2}D_{\langle\alpha}(\sigma^\mu{}_{\beta\rangle} + \omega^\mu{}_{\beta\rangle}) - \frac{1}{2}C^\mu{}_{\alpha\beta\nu}\, u^\nu$$

$${}^{(1)}\kappa^\mu{}_{\alpha\beta\gamma} = -\,(\sigma^\mu{}_{\langle\gamma} + \omega^\mu_{\langle\gamma})\sigma_{\alpha\beta\rangle}$$

} at $\mathcal{O}$

Position drift



$$\kappa^\sigma|_{\mathcal{O}} = {}^{(0)}\kappa^\sigma|_{\mathcal{O}} - \frac{{}^{(1)}\kappa^\sigma}{\mathfrak{H}}\bigg|_{\mathcal{O}} z + O(z^2)$$

$${}^{(0)}\kappa^\sigma = p^\sigma{}_\mu \, e^\alpha \, {}^{(0)}\kappa^\mu{}_\alpha$$

$${}^{(1)}\kappa^\sigma = p^\sigma{}_\mu \Big[{}^{(1)}\kappa_0^\mu + e^\alpha \, {}^{(1)}\kappa^\mu{}_\alpha + e^\alpha e^\beta \, {}^{(1)}\kappa^\mu{}_{\langle\alpha\beta\rangle} + e^\alpha e^\beta e^\gamma \, {}^{(1)}\kappa^\mu{}_{\langle\alpha\beta\gamma\rangle} \Big]$$

$$\mathfrak{H}_{\mathcal{O}} = \frac{1}{3}\theta_{\mathcal{O}} + \sigma_{\mathcal{O}\,\mu\nu} \, e^\mu_{\mathcal{O}} e^\nu_{\mathcal{O}}$$

toroidal dipole

$${}^{(0)}\kappa^\sigma{}_\mu = \sigma^\sigma{}_\mu + \omega^\sigma{}_\mu$$

$\longleftarrow$ at $\mathcal{O}$

quadrupole

$${}^{(1)}\kappa_0^\sigma = \frac{1}{6} h^{\alpha\beta} D_\alpha(\sigma^\mu{}_\beta + \omega^\mu{}_\beta) - \frac{1}{4} R^\mu{}_\nu u^\nu$$

poloidal dipole

$${}^{(1)}\kappa^\mu{}_\alpha = -\theta(\sigma^\mu{}_\alpha + \omega^\mu{}_\alpha) - (\sigma^\beta{}_\alpha + \omega^\beta{}_\alpha)(\sigma^\mu{}_\beta + \omega^\mu{}_\beta) + \frac{1}{2} R^\mu{}_\alpha + C^\mu{}_{\nu\alpha\beta} u^\nu u^\beta - \frac{4}{15} h^{\gamma\beta}(\sigma^\mu{}_\gamma + \omega^\mu{}_\gamma)\sigma_{\alpha\beta}$$

$${}^{(1)}\kappa^\mu{}_{\alpha\beta} = \frac{1}{2} D_{\langle\alpha}(\sigma^\mu{}_{\beta\rangle} + \omega^\mu{}_{\beta\rangle}) - \frac{1}{2} C^\mu{}_{\alpha\beta\nu} u^\nu$$

$${}^{(1)}\kappa^\mu{}_{\alpha\beta\gamma} = -(\sigma^\mu{}_{\langle\gamma} + \omega^\mu_{\langle\gamma})\sigma_{\alpha\beta\rangle}$$

at $\mathcal{O}$

Redshift drift

$$\xi|_{\mathscr{O}} = -\frac{{}^{(0)}\Pi|_{\mathscr{O}}}{\mathfrak{H}_{\mathscr{O}}}\, z + O(z^2)$$

$$\mathfrak{H}_{\mathscr{O}} = \frac{1}{3}\theta_{\mathscr{O}} + \sigma_{\mathscr{O}\,\mu\nu}\, e^{\mu}_{\mathscr{O}}\, e^{\nu}_{\mathscr{O}}$$

$${}^{(0)}\Pi = -\frac{1}{3} R_{\alpha\beta}\, u^{\alpha} u^{\beta} + \frac{1}{5}\sigma_{\alpha\beta}\,\sigma^{\alpha\beta} + \frac{1}{3}\omega_{\alpha\beta}\,\omega^{\alpha\beta}$$

monopole

$$+e^{\alpha} e^{\beta} \left(-C_{\alpha\gamma\beta\delta}\, u^{\gamma} u^{\delta} - \frac{1}{2} h^{\mu}{}_{\langle\alpha}\, h^{\nu}{}_{\beta\rangle}\, R_{\mu\nu} + \frac{3}{7}\sigma_{\langle\alpha}{}^{\gamma}\,\sigma_{\beta\rangle\gamma} - 2\sigma_{\langle\alpha}{}^{\gamma}\,\omega_{\beta\rangle\gamma} + \omega_{\langle\alpha}{}^{\gamma}\,\omega_{\beta\rangle\gamma}) \right)$$

quadrupole

$$-e^{\alpha} e^{\beta} e^{\gamma} e^{\sigma}\, \sigma_{\langle\alpha\sigma}\,\sigma_{\beta\gamma\rangle}$$

16-pole

What do we learn from all that?

We can combine local drifts measurements with direction-dependent Hubble law measurements

$$d_L(z, e^i) = \frac{1}{\mathfrak{H}_{\mathcal{O}}(e^i)}\, z + O(z^2)$$

$$\mathfrak{H}_{\mathcal{O}} = \frac{1}{3}\theta_{\mathcal{O}} + \sigma_{\mathcal{O}\,\mu\nu}\, e^{\mu}_{\mathcal{O}}\, e^{\nu}_{\mathcal{O}}$$

Hubble law gives $\theta_{\mathcal{O}}, \sigma_{\mathcal{O}\,\mu\nu}$

Position drift at the leading order gives $\omega_{\mathcal{O}\,\mu\nu}, \sigma_{\mathcal{O}\,\mu\nu}$

$${}^{(0)}\kappa^{\sigma}{}_{\mu} = \sigma^{\sigma}{}_{\mu} + \omega^{\sigma}{}_{\mu}$$

possibility of a consistency check for $\sigma_{\mathcal{O}}$

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$$d_L(z, e^i) = \frac{1}{\mathfrak{H}_{\mathcal{O}}(e^i)} z + O(z^2) \qquad \mathfrak{H}_{\mathcal{O}} = \frac{1}{3}\theta_{\mathcal{O}} + \sigma_{\mathcal{O}\,\mu\nu}\, e^{\mu}_{\mathcal{O}} e^{\nu}_{\mathcal{O}}$$

Hubble law gives $\theta_{\mathcal{O}}, \sigma_{\mathcal{O}\,\mu\nu}$

Position drift at the leading order gives $\omega_{\mathcal{O}\,\mu\nu}, \sigma_{\mathcal{O}\,\mu\nu}$

$${}^{(0)}\kappa^{\sigma}{}_{\mu} = \sigma^{\sigma}{}_{\mu} + \omega^{\sigma}{}_{\mu}$$

possibility of a consistency check for $\sigma_{\mathcal{O}}$

Including the redshift drift, i.e. ${}^{(0)}\xi\; \mathfrak{H}_{\mathcal{O}}$ gives

monopole $\quad -\frac{1}{3} R_{\alpha\beta} u^{\alpha} u^{\beta} + \frac{1}{5}\sigma_{\alpha\beta}\sigma^{\alpha\beta} + \frac{1}{3}\omega_{\alpha\beta}\omega^{\alpha\beta}$ — can be solved for $R_{\alpha\beta} u^{\alpha} u^{\beta}$

quadrupole $\quad -C_{\alpha\gamma\beta\delta} u^{\gamma} u^{\delta} - \frac{1}{2} h^{\mu}{}_{\langle\alpha} h^{\nu}{}_{\beta\rangle} R_{\mu\nu} + \frac{3}{7}\sigma_{\langle\alpha}{}^{\gamma}\sigma_{\beta\rangle\gamma} - 2\sigma_{\langle\alpha}{}^{\gamma}\omega_{\beta\rangle\gamma} + \omega_{\langle\alpha}{}^{\gamma}\omega_{\beta\rangle\gamma}$ — can be solved for $C_{\alpha\mu\beta\nu} u^{\mu} u^{\nu} + \frac{1}{2} R_{\mu\nu} h^{\mu}{}_{\langle\alpha} h^{\nu}{}_{\beta\rangle}$

16-pole $\quad \sigma_{\langle\alpha\sigma}\sigma_{\beta\gamma\rangle}$ — another consistency check for $\sigma_{\mathcal{O}}$

What do we learn from all that?

Information loss in the position drift due to the transverse projections

$$\kappa^\sigma|_{\mathcal{O}} = {}^{(0)}\kappa^\sigma|_{\mathcal{O}} - \left.\frac{{}^{(1)}\kappa^\sigma}{\mathfrak{H}}\right|_{\mathcal{O}} z + O(z^2)$$

$${}^{(0)}\kappa^\sigma = p^\sigma{}_\mu \, e^\alpha \, {}^{(0)}\kappa^\mu{}_\alpha$$

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Not all components of the higher multipoles of ${}^{(1)}\kappa^\mu$ can be recovered even from a perfect measurement (degeneracies)

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Not all components of the higher multipoles of ${}^{(1)}\kappa^\mu$ can be recovered even from a perfect measurement (degeneracies)

Need to perform the decomposition into vector spherical harmonics

$${}^{(1)}\kappa^B q_{AB} = \underbrace{\left(\sum a_{l,m}\,\partial_A Y_l^m\right)}_{\text{poloidal multipoles}} + \underbrace{\left(\sum b_{l,m}\,\partial_C Y_l^m\right)\ \epsilon^C{}_A}_{\text{toroidal multipoles}}$$

Check which components of the $\kappa^\mu_{\mathcal{O}}$ decomposition can be recovered from $a_{l,m}$, $b_{l,m}$

What do we learn from all that?

Information loss in the position drift due to the transverse projections

$$\kappa^\sigma\big|_{\mathcal{O}} = {}^{(0)}\kappa^\sigma\big|_{\mathcal{O}} - \frac{{}^{(1)}\kappa^\sigma}{\mathfrak{H}}\bigg|_{\mathcal{O}} z + O(z^2)$$

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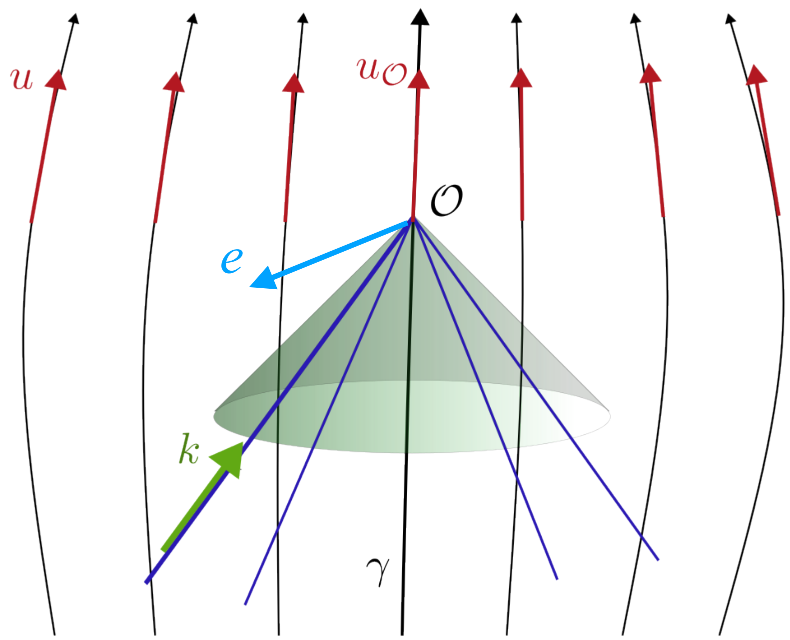
Check which components of the $\kappa^\mu_{\mathcal{O}}$ decomposition can be recovered from $a_{l,m}, b_{l,m}$

Both $\omega_{\mathcal{O}\,\mu\nu}$ and $\sigma_{\mathcal{O}\,\mu\nu}$ **can** be recovered from the 0th order ${}^{(0)}\kappa^\mu$, no information loss here

Summary, future prospects



Finite number of parameters locally describing fairly general anisotropic expansion pattern and local geometry

We derived covariant relationships between multipole decomposition coefficients and local geometry. Only finite number of multipoles at every order in z

Measurement of position drift and redshift drift, plus Hubble relation, for small z can be used to probe the anisotropic expansion and vorticity of matter flow

Also possible to recover parts of curvature (Ricci and electric Weyl)

Summary, future prospects

We also need to include the kinematics (effects of motion wrt to the cosmic flow)

In FRLW already known: [Liske *et al* 2008, Inoue *et al* 2019, Marcori *et al* 2018, Bessa *et al* 2023]

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velocity of the observer

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- Doppler boost in the redshift space

acceleration of the observer

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velocity of the source }

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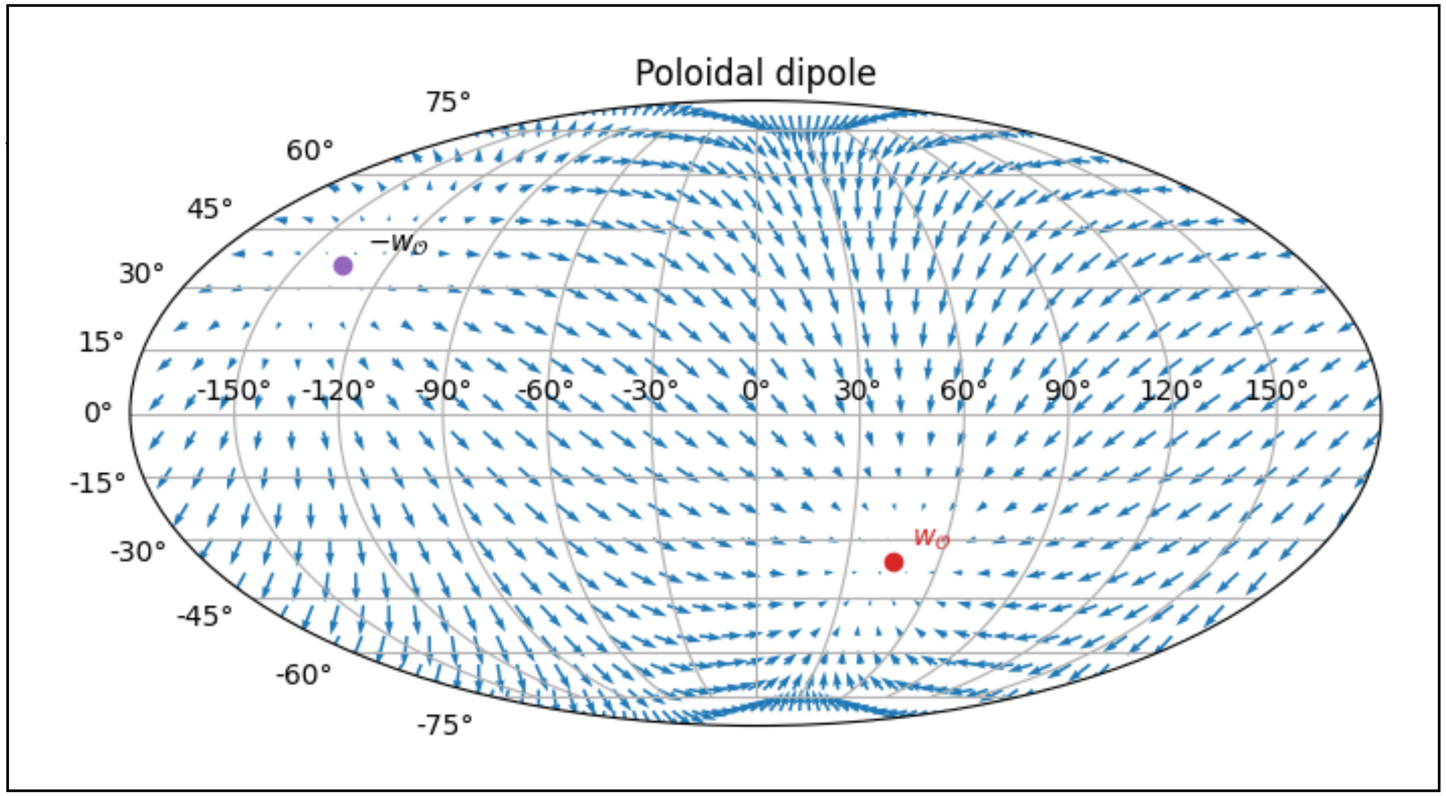

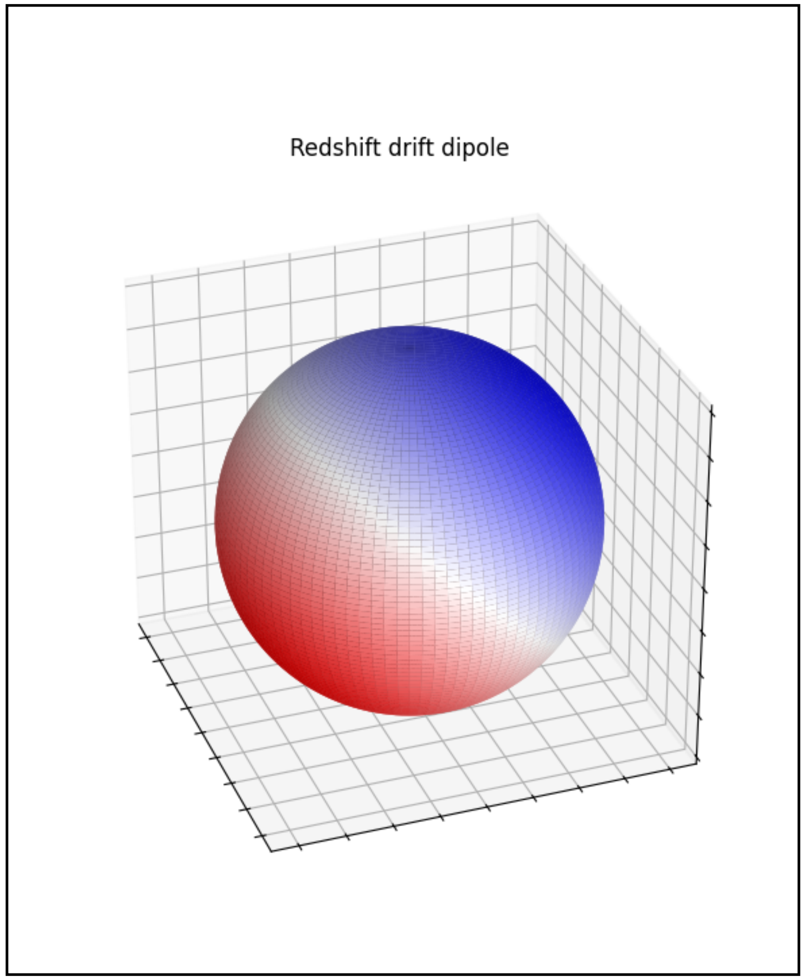

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Possible practical application: re-analyse the Gaia data for quasars, look for signs of anisotropy beyond aberration drift

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Thank you!