Quantum (near) extremal black holes

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Geometry of classical and quantum space-times
Jerzy Lewandowski Memorial Conference

A few inspiring words

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"Doing 'just math' is not a bad thing because most people are not even doing this."

Chapter 1:
Cosmological constant corrections

joint work with Gary Horowitz and Jorge Santos

A toy model

On the background of the extremal RN (AdS)

$$g = (\rho - r_+)^2 F(\rho) dv^2 + 2 dv d\rho + \rho^2 \mathring{\gamma}_{ab} dx^a dx^b, \tag{1}$$

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let us consider a massless scalar field $g^{\mu\nu}\nabla_\mu \nabla_\nu \phi = 0$. We are interested in stationary solutions. Since the background is spherically symmetric, we can separate variables: $\phi = \sum_{\ell,m} Y_{\ell m} \phi_{\ell m}(\rho)$. EOMs read:

$$0 = (\rho - r_+)^2 \rho^2 \phi_{\ell m,\rho\rho} + ((\rho - r_+)^2 \rho^2 F)_{,\rho} \phi_{\ell m,\rho} + \ell(\ell+1)\phi_{\ell m}. \tag{2}$$

Equations near the horizon

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This is a simple ODE with $\rho = r_+$ being its regular singular point. Thus, near $\rho = r_+$ we can approximate it by Euler equation:

$$0 = (\rho - r_+)^2 r_+^2 F(r_+) \phi''_{\ell m} + 2(\rho - r_+) r_+^2 F(r_+) \phi'_{\ell m} + \ell(\ell+1)\phi_{\ell m} \tag{4}$$

and so the leading order term is $\phi_{\ell m} \sim (\rho - r_+)^\gamma$, where

$$\gamma_\pm = \frac{\pm\sqrt{1 + \frac{4\ell(\ell+1)}{\sqrt{1-4Q^2\Lambda}}} - 1}{2} \tag{5}$$

and we of course choose the positive solution.

Consequences

If we choose $\ell = 1$, we get $\gamma < 1$ as long as $Q^2\Lambda < 0$. As a result:

$$T_{\rho\rho} \sim (\phi_{,\rho})^2 \sim (\rho - r_+)^{2(\gamma-1)} \to \infty \tag{6}$$

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No, because ∂_ρ is tangent to the affinely parametrized null geodesics, so this component has a well-defined interpretation! Moreover, it enters the Raychaudhuri equation, so this is definitely a physical singularity.

Setting

Since we are interested only in the near horizon behavior, we may write our (generic yet stationary) fields as $g = \mathring{g} + \delta g$, $F = \mathring{F} + \delta F$, where

$$\mathring{g} = 2\,\mathrm{d}v \left(\mathrm{d}\rho + \rho\, h_a\, \mathrm{d}x^a - \frac{1}{2}\,\rho^2\, C \mathrm{d}v \right) + q_{ab}\, \mathrm{d}x^a\, \mathrm{d}x^b \tag{8a}$$

$$\mathring{F} = E\, \mathrm{d}v \wedge \mathrm{d}\rho + \rho\, W_a\, \mathrm{d}v \wedge \mathrm{d}x^a + \frac{1}{2}\, B_{ab}\, \mathrm{d}x^a \wedge \mathrm{d}x^b, \tag{8b}$$

and $\delta g, \delta F$ vanish at the horizon (and thus by continuity are small nearby). Thus, it seems reasonable to expect that $(\delta g, \delta F)$ satisfies *linearized* Einstein-Maxwell equations on the background of $(\mathring{g}, \mathring{F})$.

Perturbations

Due to the symmetries, we may decompose our perturbations into eigenspaces of $\rho\partial_\rho - v\partial_v$. They are thus of the form

$$\delta g = \rho^\gamma \left(\delta F\, \rho^2\, \mathrm{d}v^2 + 2\,\rho\,\delta h_a\, \mathrm{d}v\, \mathrm{d}x^a + \delta q_{ab}\, \mathrm{d}x^a\, \mathrm{d}x^b\right)$$

$$\delta \mathcal{F} = \rho^\gamma \Big(\delta E\, \mathrm{d}v \wedge \mathrm{d}\rho + \rho\,\delta W_a\, \mathrm{d}v \wedge \mathrm{d}x^a + \rho^{-1}\delta Z_a\, \mathrm{d}\rho \wedge \mathrm{d}x^a + \frac{1}{2}\,\delta B_{ab}\, \mathrm{d}x^a \wedge$$

It is not hard to check that this implies

$$\delta C_{\rho a \rho b} \sim \gamma(\gamma - 1)\rho^{\gamma-2} \tag{9a}$$

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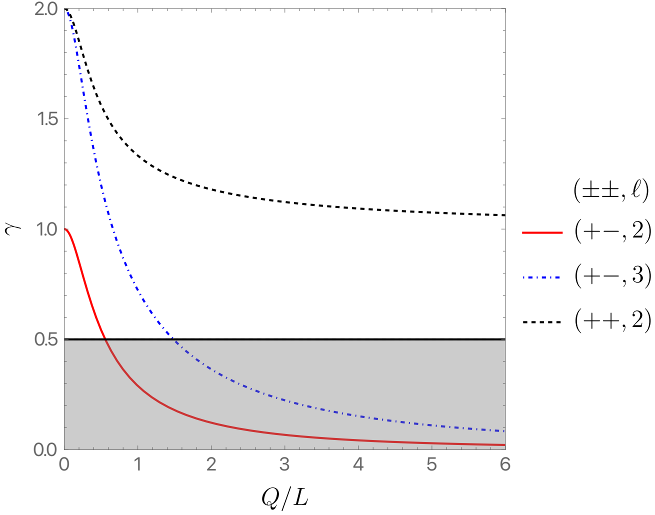
so the perturbations are singular, provided that $\gamma < 2$ (and $\gamma \neq 1$). This does not necessarily imply that those black holes are not weak solutions. This happens only when $\gamma \leq \frac{1}{2}$ [Christodoulou '09].

Exponents for RN AdS



Exponent as a function of Q and $\Lambda = -\frac{3}{L^2}$

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For $T \approx 0$, there is a huge near horizon region in which the spacetime looks like $T = 0$. In particular, we have AdS_2 factor. Thus, we expect that in this region tidal forces should have power-like growth. However, this region ends shortly before the horizon and then we go to the horizon which is located at $\rho - r_+ \sim T$.

Finite temperature

One can argue that $C_{rarb} \sim T^{\gamma-2}$ with the same exponent γ as at $T = 0$ we have $C_{rarb} \sim (\rho - r_+)^{\gamma-2}$. This is the best way to probe these exponents numerically with high accuracy.

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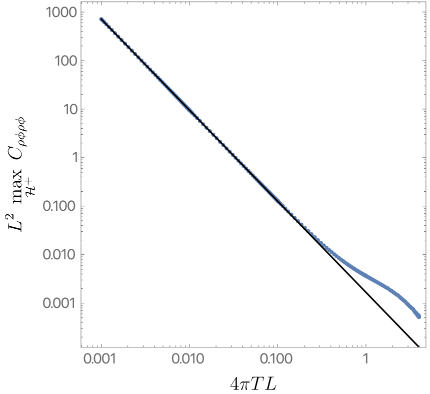

$\gamma_{numerical} \approx 0.122401$ vs $\gamma_{analytical} \approx 0.122025$

Chapter 2: Adding UV corrections

Joint work with Gary Horowitz, Grant Remmen and Jorge Santos

A (very) short introduction to EFTs

There are many reasons to believe that GR is in fact only a low-energy description of whatever the fundamental theory is. Instead of constructing UV completion, we could parametrize it as a series in derivatives (that could be apriori derived from that theory). The first non-trivial terms are

$$\mathcal{L}_4 = c_6 R^{abcd} F_{ab} F_{cd} + c_7 F_{ab} F^{ab} F_{cd} F^{cd} + c_8 F_{ab} F^{bc} F_{cd} F^{da} \tag{10}$$

We can ask what happens with the extremal Kerr-Newman for such a theory (keeping only terms linear in c_6, c_7, c_8).

Strategy

It is possible to find the EFT-corrected near-horizon limit of Kerr-Newman. Then, on this background, we may look for transversal deformations and their scaling dimensions. The whole thing can be written down as sourced linearized Einstein equations. At the end of the day, we find that the leading exponent $\gamma = 1$ gets modified.

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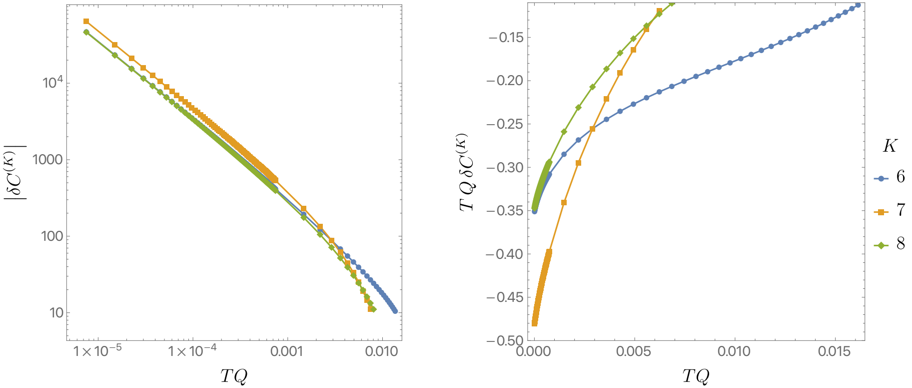
Results


$|\delta C^{(K)}|$
10^4
1000
100
10
1×10^−5
1×10^−4
0.001
0.010
TQ
$T\,Q\,\delta C^{(K)}$
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
−0.45
−0.50
0.000
0.005
0.010
0.015
TQ
K
6
7
8

Chapter 3:
Quantum gravity corrections

Joint work with Don Marolf, Ilija Rakic, Mukund Rangamani and Joaquin Turiaci

A problem

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Tree-level black hole thermodynamics for RN

$$E = E_0 + \frac{T^2}{T_q} + O(T^3), \qquad S = S_0 + \frac{T}{T_q} S_1 + O(T^2) \tag{11}$$

leads to problems for Hawking radiation. For temperatures $T \lesssim T_q$, emission of a single (neutral) quanta would lead the black hole to the overcharge. [Preskill, Schwarz, Shapere, Trivedi, Wilczek '91]

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leads to problems for Hawking radiation. For temperatures $T \lesssim T_q$, emission of a single (neutral) quanta would lead the black hole to the overcharge. [Preskill, Schwarz, Shapere, Trivedi, Wilczek '91]
Moreover, the intuition from the statistical mechanics tells us, the ground state should not have a large degeneracy unless there is a symmetry to protect it. Hence, we would expect $S(T=0) = O(1)$ at most.

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The general strategy is to work in the near-horizon region and identify zero modes associated to asymptotic symmetries of the background (being AdS_2, possibly with gauge fields).

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The general strategy is to work in the near-horizon region and identify zero modes associated to asymptotic symmetries of the background (being AdS_2, possibly with gauge fields). Then, one can use a small temperature to regulate them.

Note: this procedure is based on the assumption that nothing interesting happens outside the near-horizon region. We want to verify that assumption.

Goal

We want to calculate one-loop determinant around our favorite near-extremal background working in the full spacetime without referencing the near-horizon geometry. Moreover, we will treat temperature non-perturbatively.

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Analogously, for BTZ the near-horizon analysis predicts $Z \sim T^{2}$ (vs $T^{3/2}$ being the correct answer)

Results

The near-horizon zero modes of the extremal black hole uplift to light eigenmodes of the quadratic fluctuation operator around the near-extremal black hole saddle. The eigenvalues scale linearly with the Matsubara frequency in the near-extremal regime. They are localized at finite proper distance from the horizon.

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However, for certain backgrounds, not all zero modes get uplifted. This is in fact consistent with the near-horizon computation if one is careful enough. One can argue these modes do not contribute to $\log T$ and $\log S_0$ corrections anyway.

General framework

We want to compute the partition function

$$Z = \int \mathcal{D}X e^{-I_E[X]}. \tag{13}$$

As usual, we evaluate that expression in the saddle point expression. Around given background, we want to expand the action to the second order in fluctuations $I^{(2)}$ and write it down as

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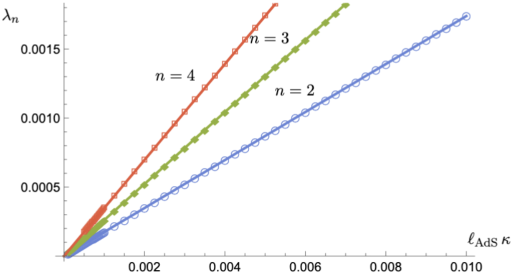
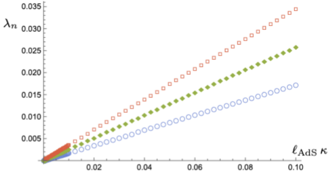
$$\det{}^{-1/2}\hat{L}. \tag{15}$$

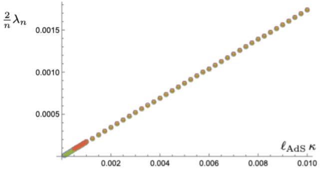
Schwarzian spectrum



a: The eigenavalue spectrum at low temperatures.

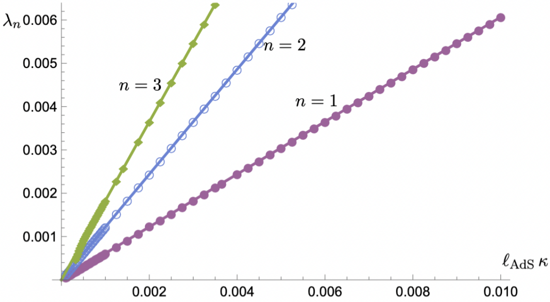


b: The eigenvalue spectrum over an extended range.



c: Rescaled eigenvalues demonstrating scaling.

$U(1)$ spectrum



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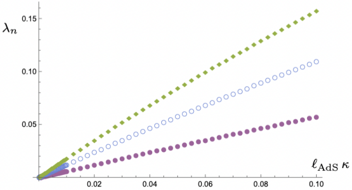

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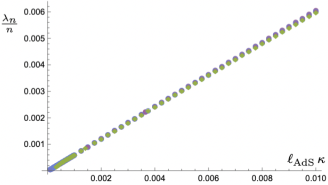

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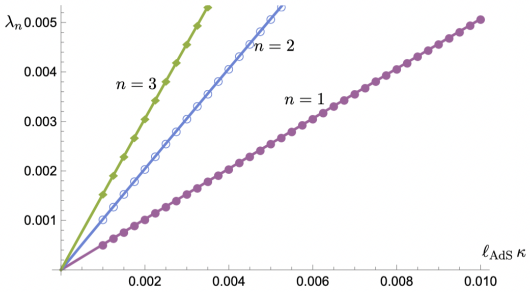
Rotational modes spectrum



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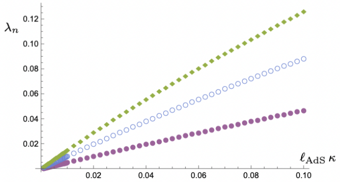

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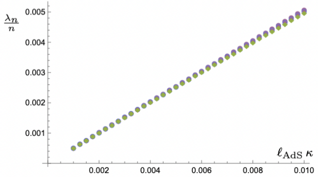

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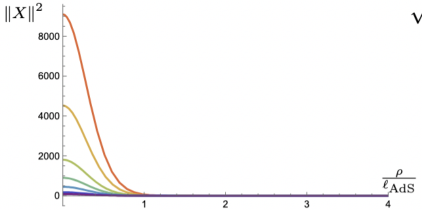
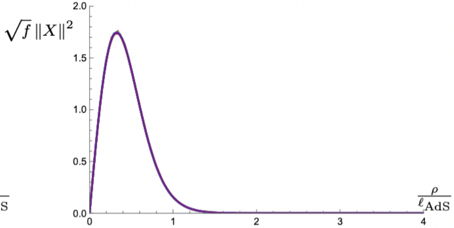
Where are they?



a: Norm against the proper distance from horizon.



b: Norm density versus proper distance from horizon

More recent developments

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$$L_0 = \frac{4\sqrt{2}\pi}{3\sqrt{21}}\sqrt{S_{dS}} \ll S_{dS} = \frac{\pi \ell^2}{G_N} \tag{16}$$

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- Near extremal Kerr-Newman with higher-curvature corrections gets large curvature near the horizon.
- The near-horizon zero mode of extremal black holes lead to light off-shell modes in a near-extremal geometry. We checked it for BTZ, RN (AdS) and hyperbolic AdS black holes.This means that one needs to take into account quantum fluctuations around these geometries.

THANK YOU FOR YOUR ATTENTION!

How close to extremality?

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Thorne's bound: $a/M < 0.998$
Cygnus X-1: $a/M > 0.9$ (details are model-dependent)
Recent observations $a/M \sim 0.9$ with $M \sim 200 M_\odot$

Rough estimates

We take c_6, c_7, c_8 to be generated by electron loops. Then, $c_7, c_8 \sim 10^{-4}(q_e/m_e)^4$ whereas $c_6 \sim 10^{-4}(q_e/m_e)^2$. For real-world electrons, we have $q_e/m_e \sim 2 \times 10^{21}$.

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$$\delta\gamma \sim c_{7/8} Z^4/G^3 M^2, \tag{17}$$

where $Z = Q/M$. We have $c_{7/8} \sim 10^{81} G^2$. The curvature scales like $\delta\gamma/T$ and so we expect

$$\delta C_{\rho\phi\rho\phi}|_H \sim 10^{81} \frac{Z^4}{GMST}. \tag{18}$$

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Let's consider a rotating black hole in an external (homogenous) magnetic field. Due to rotations, close to the horizons, charged particles will feel not only this magnetic field but also the electric one. Thus, the black hole will get charged. [Wald '74] For realistic values of magnetic fields near supermassive black holes, we have

$$Z \sim 10^{-12}. \tag{19}$$

If we consider a black hole merging with a pulsar, it may experience even larger magnetic field (for short time). The same calculation gives

$$Z \sim 10^{-7}. \tag{20}$$

So what?

$$\frac{C^{\mathcal{H}}_{\varphi\varphi} - \bar{C}^{\mathcal{H}}_{\varphi\varphi}}{\bar{C}^{\mathcal{H}}_{\varphi\varphi}} \sim (10^{-8} \ \text{to} \ 10^{-6}) \times (B/10^{16}\ \mathrm{G})^4. \tag{21}$$

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$$\frac{\mathcal{F} - \bar{\mathcal{F}}}{\bar{\mathcal{F}}} \gtrsim \text{few percent.} \tag{22}$$