On the quantum gravitational origin of MOND from quantum spin connection foam

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Precanonical quantization of pure Einsteinian vielbein gravity results in the spin connection foam (SCF) model of quantum spacetime, which is described by one- and two-point amplitudes on the spin connection bundle satisfying the space-time symmetric precanonical Schrödinger equation. The metric structure emerges as a derived quantity. The analysis of a nonrelativistic test particle in the gravitational field of a point mass, both immersed in the SCF of Minkowski spacetime, reveals a quantum modification of the Newtonian dynamics at large distances we term qMOND. A transformation to a non-inertial reference frame, defined by the mean-field acceleration arising from vacuum fluctuations of SCF, reproduces the Milgromian MOND with a theoretically derived interpolating function. The theory also establishes the relation between the Milgromian acceleration a_0 and the cosmological constant Λ: $a_0 \sim \sqrt{\Lambda}$, $\Lambda \sim (8\pi G\hbar\varkappa)^2$. Small numerical values of Λ and a_0 are linked to a hadronic scale of the parameter $\varkappa$, introduced in precanonical quantization while quantizing differential forms (e.g., $dx^1 \wedge dx^2 \wedge dx^3 \mapsto \frac{1}{\varkappa}\underline{\gamma}^0$ in $(3+1)$-dimensions) and is argued, within the theory, to be connected to the mass gap in the pure Yang-Mills sector of the Standard Model: $\Delta m \sim (g^2\hbar^4\varkappa)^{1/3}$.

1. Precanonical quantum gravity (pQG)

Precanonical quantization is based on a Dirac quantization of a generalization of Poisson brackets to a space-time symmetric generalization of the Hamiltonian formalism to field theory (the De Donder–Weyl theory) which requires no space-time decomposition.

Vielbein Einstein-Palatini Lagrangian density:

$$\mathfrak{L} = \frac{1}{8\pi G}\mathfrak{e} e_I^{[\alpha} e_J^{\beta]}(\partial_\alpha \omega_\beta^{IJ} + \omega_\alpha^{IK}\omega_{\beta K}{}^J) + \frac{1}{8\pi G}\Lambda\mathfrak{e}.$$

De Donder-Weyl Hamiltonian (DWH) formulation

$$\mathfrak{p}^\alpha_{\omega^{IJ}_\beta} := \frac{\partial \mathfrak{L}}{\partial\, \partial_\alpha \omega_\beta^{IJ}} \approx \frac{1}{8\pi G}\mathfrak{e} e_I^{[\alpha} e_J^{\beta]}, \quad \mathfrak{p}^\alpha_{e^I_\beta} := \frac{\partial \mathfrak{L}}{\partial\, \partial_\alpha e^I_\beta} \approx 0,$$

$$\mathfrak{e}H := \mathfrak{p}_\omega \partial\omega + \mathfrak{p}_e \partial e - \mathfrak{L} \approx -\mathfrak{p}^\alpha_{\omega^{IJ}_\beta}\omega_\alpha^{IK}\omega_{\beta K}{}^J - \frac{1}{8\pi G}\Lambda\mathfrak{e}.$$

⇒ Singular DWH formulation with second class constraints → generalized (Poisson-Gerstenhaber)-Dirac brackets of forms → very simple generalized Dirac brackets of fundamental variables, e.g.,

$$\{\![p^\alpha_\omega, \omega' \upsilon_\beta]\!\}^D = \delta^\alpha_\beta \delta^{\omega'}_\omega\,, \quad \{\![\mathfrak{p}^\alpha_e, e'\upsilon_{\alpha'}]\!\}^D = 0,$$
$$\{\![\mathfrak{p}^\alpha_e, \mathfrak{p}_\omega \upsilon_{\alpha'}]\!\}^D = \{\![\mathfrak{p}^\alpha_e, \omega\upsilon_{\alpha'}]\!\}^D = \{\![\mathfrak{p}^\alpha_\omega, e'\upsilon_{\alpha'}]\!\}^D = 0,$$
$$\upsilon_\alpha := \partial_\alpha \lrcorner\, dx^0 \wedge dx^1 \wedge ... \wedge dx^3.$$

Quantization $[\hat{A}, \hat{B}] = -i\hbar\widehat{\mathfrak{e}\{\![A, B]\!\}^D}$ yields

$$\widehat{\mathfrak{p}}^\alpha_{\omega^{IJ}_\beta} = -i\hbar\varkappa\mathfrak{e}\hat{\gamma}^{[\alpha}\frac{\partial}{\partial\omega^{IJ}_{\beta]}},\ \widehat{\upsilon}_\alpha = \frac{1}{\varkappa}\hat{\gamma}_\alpha,\ \hat{\gamma}^\alpha := \hat{e}^\alpha_I \underline{\gamma}^I.$$

The UV parameter $[\varkappa] = [\mathrm{cm}^{-3}]$ appears on dimensional grounds,

$$\hat{e}^\beta_I = -8\pi i G\hbar\varkappa\underline{\gamma}^J\frac{\partial}{\partial\omega^{IJ}_\beta},$$

$$\widehat{H} = 8\pi G\hbar^2\varkappa^2\ \underline{\gamma}^{IJ}\omega^{KM}_\alpha\omega_{\beta M}{}^L\frac{\partial}{\partial\omega^{KL}_\beta}\frac{\partial}{\partial\omega^{IJ}_\alpha} - \frac{1}{8\pi G}\Lambda,$$

$$\widehat{\not\nabla} = -8\pi i G\hbar\varkappa\underline{\gamma}^{IJ}\frac{\partial}{\partial\omega^{IJ}_\mu}\left(\partial_\mu + \frac{1}{4}\omega_{\mu KL}\underline{\gamma}^{KL}\overset{\leftrightarrow}{\vee}\right),$$

$$\underline{\gamma}^{IJ}\overset{\leftrightarrow}{\vee}\Psi := \frac{1}{2}\left[\,\underline{\gamma}^{IJ}, \Psi\,\right].$$

Precanonical Schrödinger equation for quantum gravity

$$\mathbf{pSE}: \quad i\hbar\varkappa\widehat{\not\nabla}\Psi = \widehat{H}\Psi \ \Rightarrow$$

$$\boxed{\underline{\gamma}^{IJ}\frac{\partial}{\partial\omega^{IJ}_\mu}\Big(\partial_\mu + \frac{1}{4}\omega^{KL}_\mu\underline{\gamma}_{KL}\overset{\leftrightarrow}{\vee} - \omega^K_{\mu M}\omega^{ML}_\beta\frac{\partial}{\partial\omega^{KL}_\beta}\Big)\Psi + \lambda\Psi = 0\,.}$$

Clifford-valued $\Psi = \Psi(\omega, x)$; $\lambda := \frac{\Lambda}{(8\pi G\hbar\varkappa)^2}$ is dimensionless, depends on the operator ordering of ω and ∂_ω.

The scalar product: $\langle\Phi|\Psi\rangle := \mathrm{Tr}\int\overline{\Phi}\,\widehat{[d\omega]}\Psi,\ \overline{\Psi} := \underline{\gamma}^0\Psi^\dagger\underline{\gamma}^0,$

$$\widehat{[d\omega]} \sim \hat{\mathfrak{e}}^{-6}\prod_{\mu IJ} d\omega^{IJ}_\mu, \quad \hat{\mathfrak{e}}^{-1}\ \text{is constructed from}\ \hat{e}^\beta_I.$$

Few consequences of pQG

⇒ The spin connection foam (SCF) picture of the geometry of quantum gravity in terms of the Clifford-algebra-valued precanonical wave function on the bundle of spin connection coefficients over spacetime, $\Psi(\omega, x) = \langle\Psi|\omega, x\rangle$, and the transition amplitudes $\langle\omega, x|\omega', x'\rangle$, the Green functions of pSE and a quantum analog of connection.

⇒ The normalizability $\langle\Psi|\Psi\rangle < \infty$ leads to the quantum-gravitational avoidance of curvature singularities by the precanonical wave function.

⇒ In the context of quantum cosmology, $\Psi(\omega, x)$ defines the statistics of local fluctuations of spin-connection, the Hubble parameter $\dot{a}/a$ classically, not the "distribution of quantum universes according to the Hubble parameter" as in the mini-superspace quantum cosmology resulting from canonical quantization of GR.

⇒ The evolution of matter/radiation on the background of quantum gravitational fluctuations whose statistics is predicted by pSE may lead to observable consequences for the distribution of matter/radiation at large cosmological scales.

2. Quantum states of Minkowski spacetime

$\eta^{\mu\nu} = (+1, -1, -1, -1) \Rightarrow \omega^{IJ}_\mu = 0 \Rightarrow$ simplifies pSE:

$$\underline{\gamma}^{IJ}\partial_{\omega^{IJ}_\mu}\partial_\mu\Psi = 0.$$

From $\langle\hat{g}^{\mu\nu}\rangle(x) = \mathrm{Tr}\int d^{24}\omega\ \overline{\Psi}(\omega, x)\hat{\mathfrak{e}}^{-6}\hat{g}^{\mu\nu}\Psi(\omega, x) = \eta^{\mu\nu}$

$$\Rightarrow \hat{g}^{\mu\nu}\Psi = -(8\pi G\hbar\varkappa)^2\eta^{IK}\eta^{JL}\partial_{\omega^{IJ}_\mu}\partial_{\omega^{KL}_\nu}\Psi = \eta^{\mu\nu}\Psi, \quad (1)$$
$$\text{and} \quad \eta^{\mu\nu}\partial_\mu\partial_\nu\Psi = 0.$$

⇒ Quantum states of (1+3)-dim Minkowski spacetime:

- light-like modes $k_\mu k_\nu = 0$ along the spacetime dims;
- 4 massive (Yukawa) modes given by (1) in $(3+3)$-dim subspaces of ω^{IJ}_μ for each $\mu = 0, 1, 2, 3$;
- the range of the massive modes in ω-space defines an invariant scale of *accelerations* $a_* = 8\pi G\hbar\varkappa$.

3. $\varkappa$ from the mass gap in pure gauge theory

Precanonical quantization of pure Yang-Mills theory ⇒

$$\widehat{H} = \frac{1}{2}\hbar^2\varkappa^2\partial^2_{A^\mu_a A^a_\mu} - \frac{1}{2}ig\hbar\varkappa C^a{}_{bc}A^b_\mu A^c_\nu\gamma^\nu\partial_{A^a_\mu}. \quad (2)$$

The spectrum of masses of propagating modes = eigenvalues of the DW Hamiltonian operator $\widehat{H}$ for

$$i\hbar\varkappa\gamma^\mu\partial_\mu\Psi = \widehat{H}\Psi, \quad \Psi = \Psi(A^a_\mu, x^\mu). \quad (3)$$

The standard functional Schrödinger equation for the wave functional $\mathbf{\Psi}([A(\mathbf{x})], t)$ can be derived from (2), (3) using the (3+1) decomposition and the "dequantization map" $\frac{1}{\varkappa}\underline{\gamma}_0 \mapsto d\mathbf{x}$. In terms of the multiple Volterra product integral over $\mathbf{x}$,

$$\mathbf{\Psi} \sim \mathrm{Tr}\prod_{\mathbf{x}}\left\{e^{\frac{\mathrm{i}}{2\varkappa}\gamma^0\gamma^{ij}A^a_0(\mathbf{x})F^a_{ij}(\mathbf{x})}\gamma^0\Psi_\Sigma(A^a_\mu(\mathbf{x}))\right\}\Big|_{\frac{1}{\varkappa}\gamma^0\mapsto\mathrm{d}\mathbf{x}}. \quad (4)$$

$$\text{For SU}(2)\ \text{theory}: \langle\frac{1}{\varkappa}\hat{H}\rangle > \left(\frac{8g^2\hbar^4\varkappa}{32}\right)^{1/3}|\mathrm{ai}'_1|, \quad (5)$$

ai'_1 is the first root of the derivative of Airy function.

⇒ From QCD mass gap $\Delta m \sim (g^2\hbar^4\varkappa)^{1/3} \sim 10^{0\pm1}\mathrm{GeV}$, and $g = g_s(Q = 0) \approx 2\pi$, and a factor $\sim 10^1$ error in the estimation (5) ⇒

$$\varkappa \sim 10^{0\pm2\times3}\,\mathrm{GeV}^3.$$

4. The cosmological constant

Weyl reordering in the 2nd term of pSE $\Rightarrow \lambda = 3$,

$$\Rightarrow \quad \Lambda = 3(8\pi G\hbar\varkappa)^2 \sim 10^{-45\pm2\times6}\ \mathrm{cm}^{-2} \quad (6)$$

originates from quantum fluctuations of spin connection. The observable Λ is obtained with $\varkappa \sim 10^{-3}\,\mathrm{GeV}^3$.

5. The minimal acceleration

With the hadronic scale of $\varkappa$, the value of

$$a_* := 8\pi G\hbar\varkappa = \sqrt{\Lambda/3} \sim 10^{-23\pm3\times2}\ \mathrm{cm}^{-1}$$

is comparable with the phenomenological Milgromian acceleration from MOND: $a_0 \approx 10^{-29}\mathrm{cm}^{-1}$.

⇒ This implies the threshold of accelerations $a_* = 8\pi G\hbar\varkappa$ emerges from quantum fluctuations of spin connection around $\omega = 0$, makes the classical notion of inertial frames not applicable below a_*.

⇒ For low accelerations below a_* the usual dynamics is modified by the quantum fluctuations of the spin connection, whose statistics is described by the pSE.

⇒ The mysterious relation from MOND [18]: $a_0 \sim \sqrt{\Lambda}$, emerges as an elementary consequence of pQG.

6. qMOND from pQG

Nonrelativistic geodesic in the fluctuating gravitational field $\tilde{\Gamma}$ of the body of mass M (static approximation)

$$\ddot{x}^i = -\tilde{\Gamma}^i_{00} = -GM\frac{x^i}{r^3} - \tilde{\omega}^i,\ \langle\tilde{\omega}^i\rangle = 0,\ \omega^i := \omega^{i0}_0 = \Gamma^i_{00}. \quad (7)$$

In the context of pQG, the values of spin connection at a point are probabilistically distributed according to the wave function $\Psi(\omega, x)$ obeying the static limit of (1)

$$\eta^{ij}\partial_{\omega^{i0}_0}\partial_{\omega^{j0}_0}\Psi = -\frac{1}{(8\pi G\hbar\varkappa)^2}\eta^{00}\Psi \quad (8)$$

whose ground state (Yukawa) solution, $\omega := \sqrt{(\omega^{0i}_0)^2}$,

$$\Psi = \frac{1}{\pi\sqrt{8G\hbar\varkappa}\,\omega}\ e^{-\omega/(8\pi G\hbar\varkappa)}, \quad \langle\overline{\Psi}|\Psi\rangle = 1. \quad (9)$$

From the average of the square of (7), ⇒ **qMOND law**:

$$a = \sqrt{\frac{G^2M^2}{r^4} + \bar{a}^2}, \quad (10)$$

where $\langle(\ddot{x}^i)^2\rangle =: a^2$, $\bar{a} := \sqrt{\langle(\tilde{\omega}^i)^2\rangle}$ is the fundamental acceleration due to the omnipresent quantum fluctuations of (static) SCF

$$\text{From (8)}: \bar{a}^2 = \int d^3\omega^i\,\overline{\Psi}\omega^2\Psi = \frac{1}{2}(8\pi G\hbar\varkappa)^2 = \frac{1}{2}a_*^2. \quad (11)$$

⇒ qMOND potential, such that $\mathbf{a} = -\nabla\Phi^{(2)}$:

$$\boxed{\Phi^{(2)}(r) = -\frac{GM}{r}\,{}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; -\frac{\bar{a}^2r^4}{G^2M^2}\right).} \quad (12)$$

At small r or vanishing $\bar{a}$, $\Phi^{(2)} \approx -GM/r$.
At large r, $\Phi^{(2)}(r) \approx \bar{a}r$ ("anti-screening" effect of SCF).

By averaging the 4th and 6th degree of (7), we get more general higher-order qMOND potentials with the same asymptotes in terms of the Appel F_1 functions,

$$\boxed{\Phi^{(4)}(r) = -\frac{\Gamma(5/4)\Gamma(1/2)}{\Gamma(7/4)}\frac{b^2}{2\bar{a}GM}\ F_1\left(\frac{5}{4}, \frac{1}{2}, \frac{3}{4}; \frac{7}{4}; z, z\right)},$$

$$z = \frac{G^4M^4 + 6G^2M^2\bar{a}^2r^4}{G^4M^4 + 6G^2M^2\bar{a}^2r^4 + b^4r^8},\ b^4 = \int d^3\omega^i\,\overline{\Psi}\omega^4\Psi,$$

and the Lauricella $F^{(3)}_D$ functions,

$$\boxed{\Phi^{(6)}(r) = -crF^{(3)}_D\left(\frac{1}{4}, -\frac{1}{6}\ -\frac{1}{6}, -\frac{1}{6}; \frac{3}{4}; \frac{s_1}{r^4}, \frac{s_2}{r^4}, \frac{s_3}{r^4}\right)},$$

$$c^6 = \int d^3\omega^i\,\overline{\Psi}\omega^6\Psi,$$

$$c^6\Pi^3_{i=1}(1 + s_it) = G^6M^6t^3 + 15G^4M^4\bar{a}^2t^2 + 15G^2M^2b^4t + c^6,$$

$$\boxed{\begin{aligned} s_1 &= -\frac{1}{-5\bar{a}^2/G^2M^2 + U + V} \\ s_2 &= -\frac{1}{-5\bar{a}^2/G^2M^2 + U\theta + V\theta^2} \\ s_3 &= -\frac{1}{-5\bar{a}^2/G^2M^2 + U\theta^2 + V\theta} \end{aligned}}$$

where
$\theta = e^{2\pi i/3}$, $\quad U = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}}$, $\quad V = \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}$,
$\Delta = (q/2)^2\ + (p/3)^3$, $\quad p = 15\big(b^4 - 5\bar{a}^4\big)/G^4M^4$,
$q = \big(250\bar{a}^6 - 75\bar{a}^2b^4 + c^6\big)/G^6M^6$.

A comparison with the following derivation of MOND from qMOND and discussions of MOND in the Solar System indicates that a higher-order $\Phi^{(2n)}$ may provide a better fit for Solar System ephemerides than $\Phi^{(2)}(r)$, which is more suitable for galaxy rotation curves.

7. MOND from qMOND

Non-inertial effects due to $\bar{a} \neq 0$ (a fictious force):

$$\frac{GM}{r^2} + \bar{a} = a \tag{13}$$

$$\Rightarrow \frac{GM}{r^2} = \sqrt{g^2 + \bar{a}^2} - \bar{a},\ g = a - \bar{a} \tag{14}$$

This has the form postulated by Milgrom in MOND (Modified Newtonian Dynamics)

$$\frac{GM}{r^2} = \mu\left(\frac{g}{a_0}\right) g\,, \tag{15}$$

provided that

$$a_0 = 2\bar{a}\,, \tag{16}$$

and the interpolating function (IF) $\mu(x)$

$$\mu(x) = \frac{1}{2x}\left(\sqrt{4x^2+1} - 1\right), \tag{17}$$

such that $\mu(x)|_{x\to\infty} \to 1$ and $\mu(x)|_{x\to 0} \to x$, interpolates between the Newtonian dynamics at $a \gg a_0$ and "deep-MOND" at $a \leq a_0$.

In contrast to the standard MOND framework, where interpolating functions are postulated to achieve optimal fits with observational data, our approach derives a theoretically motivated interpolating function that is a unique solution for the problem of a non-relativistic test particle in the gravitational field of a point mass M fixed at the origin.

$\Rightarrow$ Milgromian $a_0 = 2\bar{a} = \sqrt{2}\, 8\pi G\hbar\varkappa \approx \sqrt{\frac{2}{\lambda}}\sqrt{\Lambda}.$

8. SCF in Galaxies

Orbital motion around point mass $\mathfrak{M}$

$$v(r) = \left(\frac{G^2\mathfrak{M}^2}{r^2} + \bar{a}^2 r^2\right)^{1/4}. \tag{18}$$

A minimum at

$$r_m = \sqrt{\frac{G\mathfrak{M}}{\bar{a}}}\,, \quad v_m := v(r_m) = (2\bar{a}G\mathfrak{M})^{1/4}. \tag{19}$$

Very flat parabola in the vicinity of the minimum,

$$v(r) = (2\bar{a}G\mathfrak{M})^{1/4} + \frac{\bar{a}^2}{v_m^3}(r - r_m)^2 + O((r-r_m)^3), \tag{20}$$

The first term corresponds to the asymptotic velocity of flat rotation curves predicted by MOND and aligns with the phenomenological Baryonic Tully-Fisher relation, which connects the visible (baryonic) mass of a galaxy to the velocity in the flat region of its rotation curve.

For baryonic mass $\mathfrak{M} \sim 10^{11} M_\odot$: $G\mathfrak{M} \approx 1.5 \times 10^{-2}\,\mathrm{ly}$, $r_m \approx 5\times 10^4\,\mathrm{ly}$, $v_m \approx 0.65\times 10^{-3}$ (equivalent to 195 km/s).

With an error margin of $\pm 10\%$, the rotation velocity $v(r)$ given by equation (18) can be approximated by a flat rotation curve of $v(r) \approx 210$ km/s within the radial range of $30\,\mathrm{kly}$ to $90\,\mathrm{kly}$.

This result aligns with observed flat rotation curves of galaxies such as M31 and the Milky Way.

$\triangleright$ Approximations: Fixed central point mass $\mathfrak{M}$ and ignoring correlations in SCF.

9. SCF in the Solar System

SCF correction of about 1% to Sun's gravity at a heliocentric distance of

$$r_M = \left(0.01 \times \frac{2G^2 M_\odot^2}{\bar{a}^2}\right)^{\frac{1}{4}} \sim 3 \times 10^3\,\mathrm{au}.$$

At the current location of the Voyager 1 spacecraft (166 au from the Sun), the deviation is approximately $10^{-4}\%$.

SCF correction to Kepler's third law:

$$\left(\frac{2\pi}{T}\right)^2 = \frac{G(M_\odot + M_\oplus)}{R^3} + \bar{a}^2\,\frac{R(M_\odot + M_\oplus)}{2GM_\odot M_\oplus} + O(\bar{a}^4)\,,$$

For the Earth's orbit, where R is the semimajor axis, this results in a correction of $\sim\ 10^{-9}\%$ to the orbital period (~ -0.5 ms) and a shift in the locations of the Lagrange points.

The effects of SCF in the interior of the Solar System are weaker for higher order qMOND potentials $\Phi^{(2n)}(r)$.

10. SCF on Tabletop

$M = 1\,$kg, $GM \sim 10^{-27}$ m $\Rightarrow GM/r^2 \gg \bar{a}$ at $r \ll 1\,$m.

$\Rightarrow$ at $r = 0.1\,$m from M, the SCF correction is $\sim 10^{-2}a_0 \approx 10^{-12}$ m/s^2. For a test mass of $1\,$mg, the correction to the force is $\sim 10^{-18}$ N.

The sub-attonewton sensitivity of force sensors is already achievable. Our specific masses here correspond to Oosterkamp e.a. experiments in Leiden.

$\Rightarrow$ A potential avenue for experimental testing of quantum SCF corrections to Newtonian dynamics.

11. Conclusion

- Precanonical quantization of GR leads to a viable theory of QG (a synthesis of GR and quantum theory) that is capable of explaining already observed phenomena, such as non-Keplerian galaxy rotation curves - via theoretically deriving MOND - and accelerated expansion of the universe - via clarifying the quantum gravitational origin of the cosmological constant - and also providing realistically verifiable predictions. Precanonical quantum gravity is
  * inherently non-perturbative,
  * generally covariant,
  * background-independent (requires local fiducial Minkowski structure),
  * mathematically well-defined (yet unclarified issue with nonpositive $\mathrm{Tr}(\overline{\Psi}\Psi)$),
  * can work in any number of dimensions and metric signature, avoids the global hyperbolicity restriction,
  * does not require the "Barbero-Immirzi parameter" like in LQG,
  * both quantizes gravity and "gravitizes" quantum theory by treating spacetime variables equally.
- The effects of Λ (the simplest dark energy) and a_0 (an alternative to dark matter according to MOND), and their relation $a_0 \sim \sqrt{\Lambda}$ can be understood as manifestations of SCF in pQG.
- Realistic numerical values of Λ and a_0 are obtained for a hadronic scale of $\varkappa$, which is consistent with its derived relation to the scale of the mass gap in the pure quantum YM sector of the Standard Model.
- A non-relativistic test particle within gravitating mass M immersed in the static non-relativistic approximation of SCF yields qMOND potentials with linear asymptotes that match cosmological scale slopes.
- MOND with a theoretically derived interpolating function is recovered in the non-inertial frame of the mean field $\bar{a}$ of quantum fluctuations in SCF.
- Relaxing the fixed central mass approximation and taking into account quantum correlations due to $\langle\omega_1, \mathbf{x}_1, t|\omega_2, \mathbf{x}_2, t\rangle \neq 0$ are work in progress.
- Flat galaxy rotation curves are accommodated by both MOND and (approximately) qMOND descriptions.
- The linear asymptotes of $\Phi^{(2n)}(r)$ potentials lead to improved early structure formation.
- Realistic prospects exist for laboratory and space tests of SCF quantum gravity corrections.

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