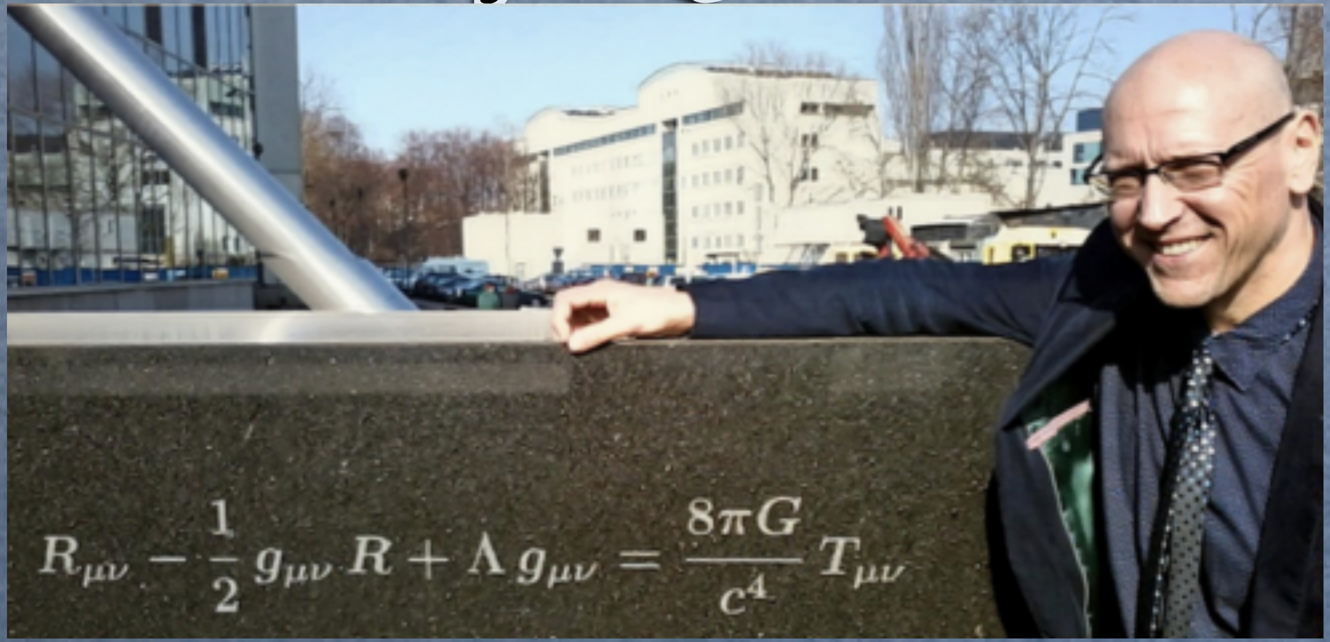
Gravity Quantised



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FAU Friedrich-Alexander-Universität Erlangen-Nürnberg

ecap ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

joint work with and Michael Kobler & Max Fahn

and Alba Domi, Thomas Eberl, Max Fahn, Lukas Hennig, Uli Katz, Roman Kemper, Michael Kobler

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Kristina Giesel, Institute for Quantum Gravity, Theoretical Physics III, FAU Erlangen-Nürnberg

I. Gravity Quantised: Loop Quantum Gravity with a Scalar Field

Gravity Quantized: Loop Quantum Gravity with a Scalar Field

Marcin Domagała,[1,*] Kristina Giesel,[2,3,†] Wojciech Kamiński,[1,‡] and Jerzy Lewandowski[1,4,§]

[1] *Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, ul. Hoża 69, 00-681 Warszawa (Warsaw), Polska (Poland)*

[2] *Excellence Cluster Universe, Technische Universität München, Boltzmannstr. 2, 85748 Garching, Germany*

[3] *Department of Physics and Astronomy, Louisiana State University, Baton Rouge LA 7080 3, United States*

[4] *Institute for Gravitation and the Cosmos, Physics Department, Penn State, University Park, PA 16802, U.S.A.*

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Abstract ..."but we do not have quantum gravity." This phrase is often used when analysis of a physical problem enters the regime in which quantum gravity effects should be taken into account. In fact, there are several models of the gravitational field coupled to (scalar) fields for which the

I. Gravity Quantised

Idea in canonical LQG: Apply (canonical) quantisation to general relativity

Lagrangian formulation: $g_{\mu\nu}, \Phi^J$ $$G_{\mu\nu} = \kappa T_{\mu\nu}$$

Geometry & matter

Differences & challenges compared to QM

I.1. GR is a (fully) constrained system, involves gauge dof [Status Report: Ashtekar, Lewandowski '05]

→ quantisation of constrained systems more ambiguities

I.2. Even at classical level physical observables (gauge-inv. quantities) hard to construct

→ (Quantum) Dirac observables with respect to physical dynamical (quantum) reference frames [Domagala, K.G., Kaminski, Lewandowski `10][Kaminsiki, Lewandowski, Pawłowski '09]

I.3. Metric becomes operator requires background independent quantisation

→ LQG no Fock representation, choice of representation [Ashtekar, Lewandowski+ Rovelli, Smolin '90] [LOST & F Theorem '05]

I.4. Formulation of the dynamics: Quantum Einstein Equations

[Ashtekar, Lewandowski, Marolf, Mourao, Thiemann '95] [Lewandowski, Marolf '97]

Many people contributed, here only some of Jurek's contributions are highlighted

Observer dependent (quantum) field theory:
Canonical quantization programme can be completed in these models

[Kuchar, Torre `91] [Rovelli, Smolin `93] [Brown, Kuchar `95] [Kuchar, Romano`95]
[Bicak, Kuchar `97] [Thiemann `06] [K.G., T. Thiemann `10]
[Domagala, K.G., Kaminski, Lewandowski `10] [Husain, Pawlowski `11] [K.G., T. Thiemann `12,]
[K.G., Vetter`19] + several applications in symmetry reduced models

Use matter reference field(s): dust or scalar field(s): simplicity of observable algebra; either one or four reference fields

Often mixture of Dirac and reduced phase space quantisation

Quantum Einstein Equations with respect to chosen reference frame at the level of physical Hilbert space

In full LQG: dynamics technically complicated

Physical Implications: symmetry reduced models: quantum and/or effective models: LQC and quantum black holes [Talks by Cafaro, Ma and Zhang]

II. Are there further physical situation where we can apply these techniques?

II.a Inspiration from Phenomenology

Next to LQC and quantum black holes, are there further physical situation where we can apply these techniques?

Oscillation probability for neutrinos

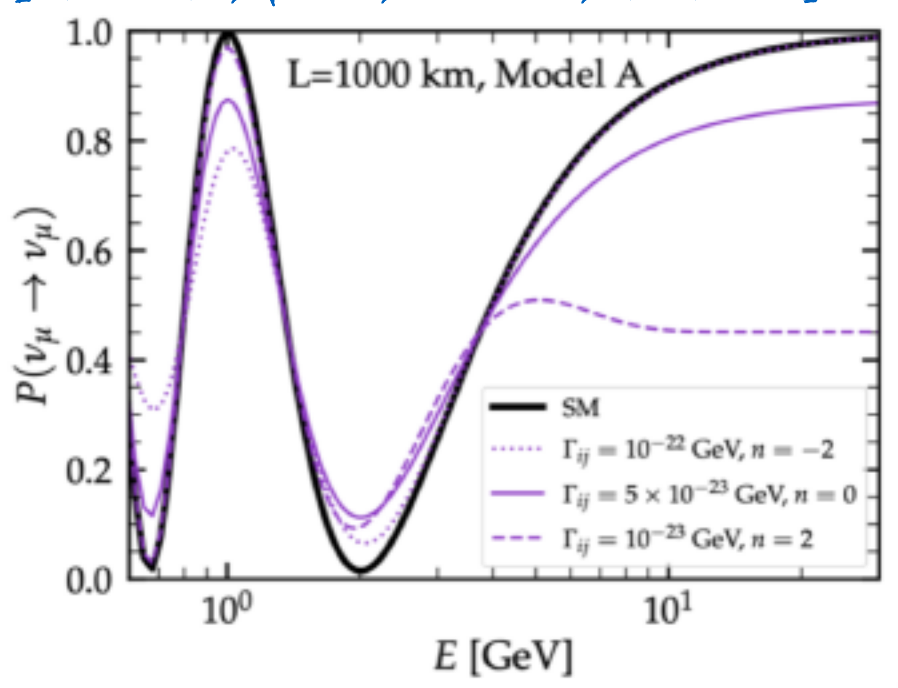
Decoherence parameters

$$P\left(\nu_\alpha \to \nu_\beta\right) = \mathrm{Tr}\left[\hat{\rho}^{(\alpha)}(t)\hat{\rho}^{(\beta)}(0)\right] = \sum_{i,j} \tilde{U}_{\alpha i}\tilde{U}^*_{\beta i}\tilde{U}^*_{\alpha j}\tilde{U}_{\beta j} e^{-\frac{i}{\hbar}(\tilde{E}_i - \tilde{E}_j)t - \Gamma_{ij} t}$$

$$\Gamma_{ij}(E) = \gamma_{ij} E^n$$

Additional damping in oscillation probability due to decoherence

[De Romeri, Giunti, Stuttard, Ternes '23]



Probabilities for oscillations change depending on the values for γ_{ij} and choice of power n

[Ellis, Lopez, Mavromatos, Nanopoulos 1996]; [Benatti, Floreanini 1999]; [Lisi, Marrone, Montanino 2000]; [Guzzo, de Holanda, Oliveira 2016]; [Gomes, Forero, Guzzo, de Holanda, Oliveira 2019]]

Not an effect observed but current experiments constrain decoherence parameters

If we consider the “Gravity Quantised Programme" can we gain some insights on these phenomenological models? What is Γ_{ij} ?

(i) Models use framework of open quantum systems

⟶ Need to transfer techniques to these systems

(ii) From QG perspective: decoherence caused by gravity interesting

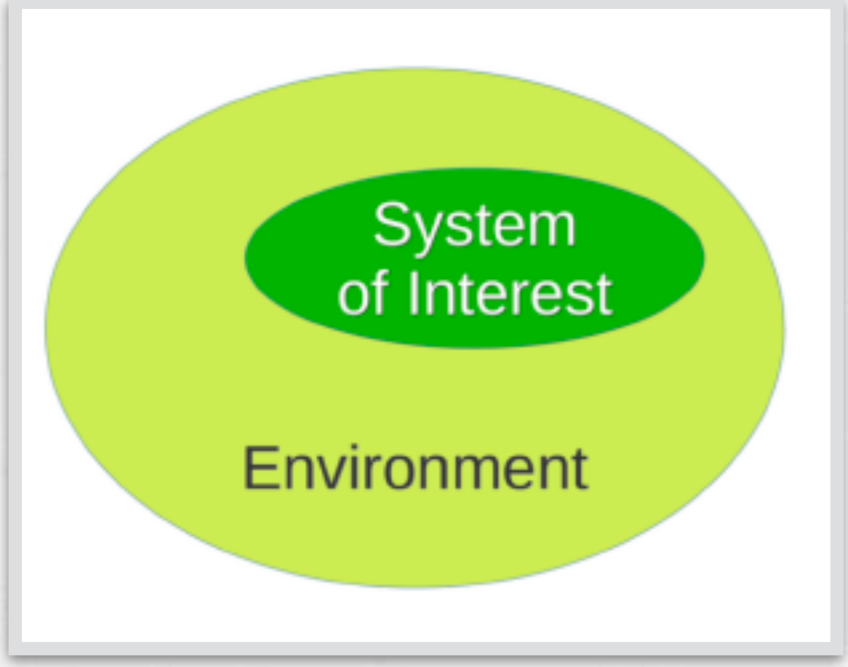
⟶ gravitationally induced decoherence [M. Fahn, K.G, M. Kobler '22] [M. Fahn, K.G. '24]

(iii) Motivation: use methods to derive decoherence parameters from underlying models and bridge to phenomenological models

II.b Brief Review on Open Quantum Models

II.b Open Quantum Systems

Aim: Consider gravitationally induced decoherence



System S + environment ε : $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_\varepsilon$

Dynamics: $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_S + \mathrm{H}_\varepsilon + \mathrm{H}_{\mathrm{int}}$

$$\mathrm{H}_{\mathrm{int}}(\mathrm{t}) = \alpha \sum_j \mathrm{S}_j(\mathrm{t}) \otimes \mathrm{E}_j(\mathrm{t})$$

Total Dynamics:

$$\partial_t \rho = \frac{1}{\mathrm{i}\hbar}[\mathrm{H}_{\mathrm{tot}}, \rho]$$

Gravitational environment interesting because all matter couples to gravity

Allows to investigate effective influence of gravity on matter systems

Requires weak coupling to environment: linearised gravity

II.b Open Quantum Systems

Total dynamics is usually to complicated to solve

Microscopic model: $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_S + \mathrm{H}_\varepsilon + \mathrm{H}_{\mathrm{int}}$ $\quad \mathrm{H}_{\mathrm{int}}(\mathrm{t}) = \alpha \sum_j \mathrm{S}_j(\mathrm{t}) \otimes \mathrm{E}_j(\mathrm{t})$

Aim: Effective dynamics for S: master equation

$$\partial_t \rho_S = \frac{\mathrm{d}}{\mathrm{dt}} \mathrm{tr}_\varepsilon \left(\mathrm{U}_{\mathrm{tot}}(\mathrm{t}) \rho(0) \mathrm{U}^\dagger_{\mathrm{tot}}(\mathrm{t}) \right) = \frac{1}{\mathrm{i}\hbar} \mathrm{tr}_\varepsilon \left([\mathrm{H}_{\mathrm{tot}}, \rho] \right)$$

Master equation encodes interaction with the environment

$$\frac{\partial \rho_S(t)}{\partial t} = -\frac{i}{\hbar} \left[H_S(t) + H_{add}(t), \rho_S(t) \right] + \mathcal{D} \left[\rho_S(t) \right]$$

Environmental correlation functions

contribution due to interaction with environment

Lindblad equation

$$\frac{\partial}{\partial t} \rho_S(t) = \frac{1}{i\hbar} \left[H_S + H_{\mathrm{LS}}, \rho_S(t) \right] + \sum_k \gamma_k \left(L_k \rho_S(t) L_k^\dagger - \frac{1}{2} \left\{ L_k^\dagger L_k, \rho_S(t) \right\} \right)$$

For given H_{S} model characterised by choice of L_k, γ_k ← QG

II.1. Linearised Gravity as an Open QFT Model

[Max Fahn, K.G., Roman Kemper, Michael Kobler '22+'25 (soon)] Ashtekar-Barbero variables
Relational formalism

[[Anastopoulos, Hu 2013]; [Blencowe 2013]; [Oniga, Wang 2016];
[Lagouvardos, Anastopoulos 2021]; [Asprea 2021]] ADM variables, gauge fixing

II.1. Matter Coupled to Linearised Gravity

Microscopic derivation: First steps in a given model

Existing work in ADM variables: [[Anastopoulos, Hu 2013]; [Blencowe 2013]; [Oniga, Wang 2016]; [Lagouvardos, Anastopoulos 2021]; [Asprea 2021]]

Scalar field coupled linearised gravity in Ashtekar-Barbero variables

$$S[A_a^j, \phi] = \int_{\mathcal{M}} \sqrt{|\det(g)|} R + S_\phi$$

linearize gravitational part in Hamiltonian formulation

$$\text{system}: \ \phi, \pi \qquad \text{environment} \ : \delta A_a^i, \delta E_a^i$$

In field theory interaction given $\frac{\kappa}{2} \int_M d^4x \delta h^{\mu\nu} T_{\mu\nu}[\Phi, \eta]$

Need true Hamiltonian system: reduced quantisation, geometrical clocks, classical dynamical reference frame

II.2. Dynamical Reference Frames: Geometrical Clocks

II.2. Geometrical dynamical reference frame

[Rovelli '90, Dittrich '04, Thiemann'04, Pons et al.]

Here we use relational formalism in a perturbative scheme

Linearised constraints: $\delta\mathcal{C}_I := (\delta C, \delta C_a, \delta G_j) \quad \delta A_a^i, \delta E_i^a$

Choose geometrical clocks such that (use dual observable map) [Fan, K.G., Kobler '22]

$$\{T^I(x), \mathcal{C}'_J(y)\} = \{\delta T^I(x), \delta\mathcal{C}'_J(y) + \delta^2\mathcal{C}'_J(y)\} = \frac{1}{\kappa}\delta^I_J \delta^{(3)}(x,y) + O\left(\delta^2, \kappa^2\right)$$

ADM-like clocks extended to Ashtekar-Barbero case and matter [ADM '60, Dittrich '06]

$$\delta T(\vec{x},t) := -\frac{1}{\kappa\beta}\left[\frac{1}{2}\delta^i_c \epsilon_a{}^{cb}\partial_b\left(\delta E^a{}_i * G^\Delta\right)(\vec{x},t) + \delta^a_i\left(\delta A_a{}^i * G^\Delta\right)(\vec{x},t)\right],$$

$$\delta\Xi^i(\vec{x},t) := \frac{2}{\kappa}\partial^a\left(\delta A_a{}^i * G^\Delta\right)(\vec{x},t),$$

$$\delta T^a(\vec{x},t) := \frac{2}{\kappa}\left(\delta^a_b\delta^i_c\partial^c - \frac{1}{2}\delta^i_b\partial^a + \delta^{ac}\delta^i_c\partial_b\right)\left(\delta E^b_i * G^\Delta\right)(\vec{x},t)$$
$$+ \frac{4\beta}{\kappa^2}\left[\frac{1}{2}\delta^i_b\partial^a\partial^b\left(\delta G_i * G^{\Delta\Delta}\right)(\vec{x},t) - \delta^{ab}\delta^i_b\left(\delta G_i * G^\Delta\right)(\vec{x},t)\right] + \frac{1}{\kappa^2}\partial^a\left[(\delta C - \kappa\epsilon) * G^{\Delta\Delta}\right](\vec{x},t)$$

This allows to construct the corresponding physical Hamiltonian describing the total dynamics of system and environment [K.G, Kabel, Wieland '24 relation to covariant phase space reference frames]

II.2. Observable map and dual map

Usual observable map (omitted integrals here)

$$\mathcal{O}_{f,\{T\}}\left(\tau^I\right) = \left[\exp\left(\xi^I\left\{C_I,\cdot\right\}\right)\cdot f\right]\Big|_{\xi^I:=T^I-\tau^I}$$
$$= f + \left(T^I-\tau^I\right)\left\{C_I,f\right\} + \frac{1}{2!}\left(T^I-\tau^I\right)\left(T^J-\tau^J\right)\left\{C_J,\left\{C_I,f\right\}\right\} + \cdots$$

Dual map: role of constraints and clocks are interchanged

Useful for constructing suitable reference frame, combination of both maps yields directly STT dof as physical dof in vacuum case

In non-vacuum case was useful to understand self-energy term and simplicity of the algebra of the Dirac observables [Fahn, K.G. Kemper to appear soon]

Relational open quantum systems: open QFT with respect to some reference frame, observer dependent also the interaction to the environment

II.2. Classical Model: Gravitationally induced decoherence

Recap: started with $((\delta A_a^j), (\delta E_j^a), (\phi, \pi))$: 9+1 dof or 18+2 on phase space

Now: $((\delta A_a^j)^{\mathrm{GI}}, (\delta E_j^a)^{\mathrm{GI}}), (\phi^{\mathrm{GI}}, \pi^{\mathrm{GI}}) + (\delta T, \delta C), (\delta T^a, \delta C_a), (\delta \Xi^j, \delta G_j)$

Degrees of freedom: physical: 2+2, 1+1 and unphysical: 7+7 Kuchar decomposition

Open quantum perspective: QFT model with determined interaction

$$\text{System} : \phi^{GI}, \pi^{GI} \quad \text{environment} : (\delta A_a^j)^{GI}, (\delta E_j^a)^{GI})$$

Physical Hamiltonian: classical model

$$\delta \mathbf{H} = \mathcal{H}_\phi + O_{\mathcal{H}_{\mathrm{geo}}, \{T\}} + O_{\mathcal{H}_I, \{T\}} \quad \text{with} \quad \mathcal{H}_{\mathrm{geo}} := \int_\sigma d^3x \delta^2 C^{\mathrm{geo}}(x)$$

System env. interaction

$$\mathcal{H}_\phi := \int_\sigma d^3x \epsilon(\vec{x}, t)$$

Standard Hamiltonian model no constraints left

II.3+II.4 Choice of Quantisation and Master Equation (Dynamics)

II.3+4 Gravitationally induced decoherence

Microscopic derivation: First steps in a given model

Assumptions of the model [Max Fahn, K.G., Michael Kobler '22]

[[Nakajima 1958]; [Zwanzig 1960]; [Shibata, Takahashi, Hashitsume 1977]; [Chaturvedi, Shibata 1979]]

Starting point: Time-ConvolutionLess (TCL) equation truncated at second oder

Assume thermal state for the gravitational environment

As first step: Fock quantisation of physical Hamiltonian and interaction

Scalar field and photon analogue form of master equation $j_r^b(\vec{k},\vec{l})$ different

system operators

Resulting master equation

$$\frac{\partial}{\partial t}\rho_S(t) = -i\left[H_S + \kappa U + \kappa H_{LS}, \rho_S(t)\right] + \mathcal{D}_{\rm first}\,\left[\rho_S\right]$$

$$\mathcal{D}_{\rm first}\,\left[\rho_S\right] := \frac{\kappa}{2}\int \frac{d^3k d^3p d^3l}{(2\pi)^{\frac{6}{2}}}\sum_{r;ab} R_{ab}(\vec{p},\vec{l};\vec{k},t)\left(j_r^b(\vec{k},\vec{l})\rho_S(t)j_r^a(\vec{k},\vec{p})^\dagger - \frac{1}{2}\left\{j_r^a(\vec{k},\vec{p})^\dagger j_r^b(\vec{k},\vec{l}),\rho_S(t)\right\}\right)^{\square}$$

not of Lindblad type [confirm results of [Oniga, Wang 2016], [Anastopoulos, Hu 2013] and [Lagouvardos, Anastopoulos 2021]]

II. Applications: One particle master equation

V. One particle master equation

One particle equation and interplay with renormalisation

[Max Fahn, K.G. '24] [Burrage, Kähding,,Millington, Minar '19]

Projection: one particle density matrix

$$\rho_1(t) = \int_{\mathbb{R}^3} d^3u \int_{\mathbb{R}^3} d^3v \rho(\vec{u}, \vec{v}, t) a_u^\dagger |0\rangle\langle 0| a_v$$

One particle master equation

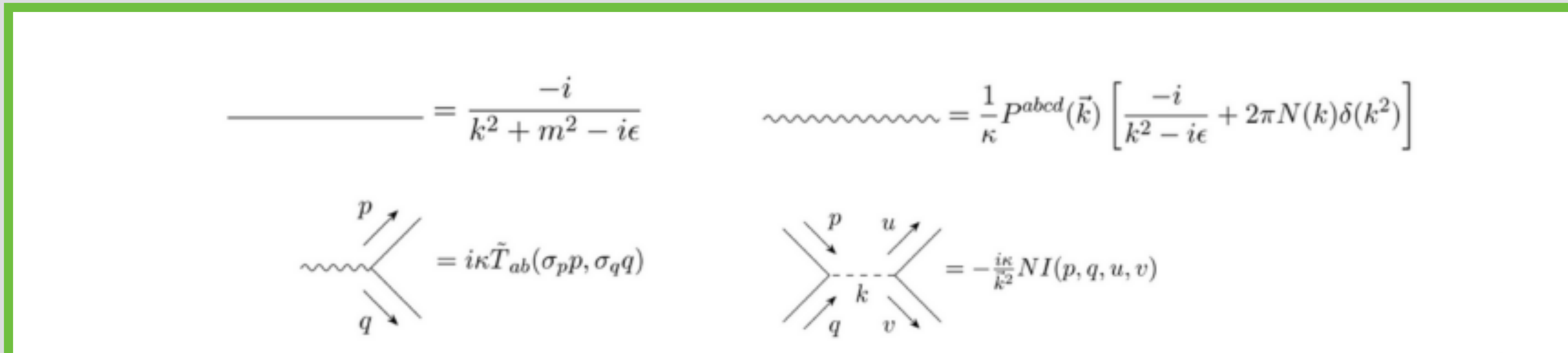
$$\begin{aligned}
\frac{\partial}{\partial t}\rho(\vec{u},\vec{v},t) = &- i\rho(\vec{u},\vec{v},t)\left(\omega_u - \omega_v\right) \\
&- \frac{\kappa}{2}\int \frac{d^3k}{(2\pi)^3}\left\{\frac{P_u(\vec{k})}{\omega_{u-k}\omega_u}\left[C(\vec{u},\vec{k},t) + \delta_P C_P(\vec{u},\vec{k},t)\right]\right. \\
&\left. + \frac{P_v(\vec{k})}{\omega_{v-k}\omega_v}\left[C^*(\vec{v},\vec{k},t) + \delta_P C_P^*(\vec{v},\vec{k},t)\right]\right\}\rho(\vec{u},\vec{v},t) \\
&+ \frac{\kappa}{2}\int \frac{d^3k}{(2\pi)^3}\frac{P^{ij\,ln}(\vec{k})u_i u_j v_l v_n}{\sqrt{\omega_{u+k}\omega_u\omega_{v+k}\omega_v}}\left\{C(\vec{u}+\vec{k},\vec{k},t) + C^*(\vec{v}+\vec{k},\vec{k},t)\right\}\rho(\vec{u}+\vec{k},\vec{v}+\vec{k},t)
\end{aligned}$$

Coefficients: $C(\vec{u},\vec{k},t) = \int_0^{t-t_0} \frac{d\tau}{\Omega_k}\left\{[N(k)+1]e^{-i(\Omega_k+\omega_{u-k}-\omega_u)\tau} + N(k)e^{i(\Omega_k-\omega_{u-k}+\omega_u)\tau}\right\}$

IV. One particle master equation

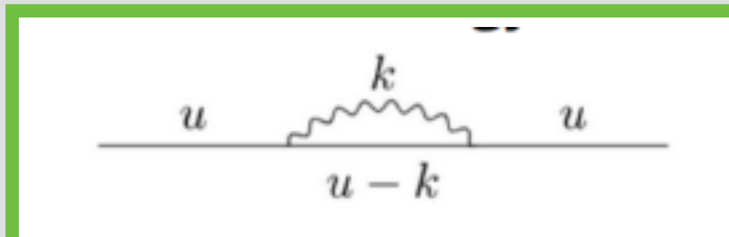
Strategy: perform renormalisation before any approximations are applied

Involved Feynman diagrams in 2nd order TCL equation [Fahn. K.G. '24]



UV divergent terms: vacuum self-energy

Renormalised one particle master equation



Consider Markov & rotating wave approximations + ultra-relativistic limit: toy model

$$\frac{\partial}{\partial t}\hat{\rho}(t) = -i[\hat{H}, \hat{\rho}(t)] + \frac{\kappa}{6\pi\beta}\left(\hat{H}\hat{\rho}(t)\hat{H} - \frac{1}{2}\left\{\hat{H}^2, \hat{\rho}(t)\right\}\right)$$

[Blencowe, Xu '22]
[Fahn, K.G. '24]
[Domi, Ebert, Fahn, K.G., Hennig, Katz, Kemper '24]

IV. Comparison with phenom. models

Taking these QM toy model inspired by the open QFT model can we match to phenomenological models?

Phenol models (PQD)

$$\tilde{\rho}_{ij} = \tilde{\rho}_{ij}(0)e^{-\frac{i}{\hbar}(\tilde{E}_i-\tilde{E}_j)t-\Gamma_{ij}(E)t}$$

$$\Gamma_{ij}(E) = \gamma_{ij}E^n \longleftarrow$$

Gravit. induc. decoh. (GQD)

$$\widetilde{\rho}_{ij}(t) = \widetilde{\rho}_{ij}(0)\cdot e^{-\frac{i}{\hbar}(\widetilde{H}_i-\widetilde{H}_j)t-\frac{4\eta^2 k_B T}{\hbar^3}(\widetilde{H}_i-\widetilde{H}_j)^2 t}$$

$$\frac{4\eta^2 k_B T}{\hbar^3}\left(\widetilde{H}_i - \widetilde{H}_j\right)^2 t \qquad \eta, T$$

η, T coupling constant and Temperatur of environment

For oscillations in vacuum exact match

$$\Gamma_{ij}(E) = \gamma_{ij}E^{-2} \quad \gamma_{ij} = \frac{\eta^2 c^8 k_B T}{\hbar^3}\left(\Delta m_{ij}^2\right)^2 \quad \text{and} \quad n = -2$$

For oscillations in matter matching not possible if constant decoherence parameters assumed that do not include matter effects [Carpio, Massoni, Gargo '18]

Existing bounds for constant Γ_{ij} cannot be used to constrain η, T

Summary & Conclusions

→ **Summary**

For general relativistic open quantum systems additional tasks:
Hamiltonian formulation involves constraints

For LQG: special choice of variables: Ashtekar-Barbero
Access physical sector via reference frames: here geometrical clocks

Linearised gravity as on open QFT model

Derived master equation for field theory model: scalar field + gravity

Applications: renormalisation of 1-particle master equation + decoherence in neutrino oscillations (bridge to phenomenological models)

→ **Outlook** *[work in progress at FAU]*

Work more towards LQG inspired models for grav. decoherence QM toy model

If phenomenological models are constrained from experimental side, what does it mean for microscopic models?

Do LQG inspired models have characteristic properties?

Thank you!