On the gravitational collapse in Loop Quantum Gravity

Luca Cafaro, University of Warsaw

Jerzy Lewandowski Memorial Conference - Warsaw, September 15–19, 2025

UNIVERSITY OF WARSAW

The Lemaitre-Tolman-Bondi model

$$\mathrm{d}s^2 = -N^2(t,R)\mathrm{d}t^2 + \frac{(\partial_R r)^2}{1+\varepsilon(t,R)}\mathrm{d}R^2 + r^2(t,R)\mathrm{d}\Omega^2$$

LTB coordinates:

1. Spatial coordinates are comoving with the fluid:
   $u^\mu \propto \delta^\mu_0$

2. The $\mathrm{t} = \mathrm{const}$ hypersurfaces are orthogonal to the flow of the fluid:
   $u_\mu \propto \partial_\mu \mathrm{t}$

3. $\mathrm{N} \equiv$ *Lapse,* $\mathrm{r} \equiv$ *areal radius* and $\varepsilon > -1$ is an arbitrary function

The Lemaitre-Tolman-Bondi model

$$\mathrm{d}s^2 = -N^2(t,R)\mathrm{d}t^2 + \frac{(\partial_R r)^2}{1+\varepsilon(t,R)}\mathrm{d}R^2 + r^2(t,R)\mathrm{d}\Omega^2$$

Anisotropic fluid $T^{\mu}{}_{\nu} = \rho u^{\mu} u_{\nu} + \Pi\, e^{\mu} e_{\nu} + \Sigma\left(\delta^{\mu}_{\nu} + u^{\mu} u_{\nu} - e^{\mu} e_{\nu}\right)$:

1. u^{μ} is the 4-velocity of the fluid
2. e^{μ} is a unit spacelike vector defining the direction of the anisotropy
3. $\rho(\mathrm{t},\mathrm{R})$, $\Pi(\mathrm{t},\mathrm{R})$ and $\Sigma(\mathrm{t},\mathrm{R})$ are respectively the energy density, radial pressure, and tangential pressure in the rest frame of the fluid

The Lemaitre-Tolman-Bondi model

$$\dot{r}^2 = N^2\left(\frac{2Gm}{r} + \varepsilon\right)$$

$$\dot{m} = -\,4\pi\Pi\, r^2\dot{r}$$

$$\dot{\varepsilon} = \frac{(1+\varepsilon)}{r'}\frac{2\dot{r}}{\rho+\Pi}\left[\frac{2r'}{r}(\Sigma-\Pi) - \Pi'\right]$$

$$\frac{N'}{N} = \frac{1}{\rho+\Pi}\left[\frac{2r'}{r}(\Sigma-\Pi) - \Pi'\right]$$

$$\dot{\Pi} = \frac{\delta\Pi}{\delta\rho}\dot{\rho}$$

$$\dot{\Sigma} = \frac{\delta\Sigma}{\delta\rho}\dot{\rho}$$

[Lasky, Lun (2006)]

The Lemaitre-Tolman-Bondi model

$$\dot{r}^2 = N^2\left(\frac{2Gm}{r} + \varepsilon\right) \quad \Longrightarrow \quad \frac{\varepsilon}{2} = \frac{\dot{r}^2}{2} - \frac{Gm}{r}$$

$$\dot{m} = -4\pi\Pi\, r^2\dot{r}$$

$$\dot{\varepsilon} = \frac{(1+\varepsilon)}{r'}\frac{2\dot{r}}{\rho+\Pi}\left[\frac{2r'}{r}(\Sigma - \Pi) - \Pi'\right]$$

$$\frac{N'}{N} = \frac{1}{\rho+\Pi}\left[\frac{2r'}{r}(\Sigma - \Pi) - \Pi'\right]$$

$$\dot{\Pi} = \frac{\delta\Pi}{\delta\rho}\dot{\rho}$$

$$\dot{\Sigma} = \frac{\delta\Sigma}{\delta\rho}\dot{\rho}$$

[Lasky, Lun (2006)]

The framework – Symmetry reduction

Hamiltonian theory in Ashtekar's variables

- $\mathcal{H}[E_i^a; A_a^i]$
- $\mathcal{H}_{\mathrm{i}}\left[E_i^a; A_a^i\right]$
- $\mathcal{G}_{\mathrm{i}}\left[E_i^a; A_a^i\right]$

The framework – Symmetry reduction

Hamiltonian theory in Ashtekar's variables

- $\mathcal{H}[E_i^a; A_a^i]$
- $\mathcal{H}_\mathrm{i}[E_i^a; A_a^i]$
- $\mathcal{G}_\mathrm{i}[E_i^a; A_a^i]$

Spherical symmetry →

- $\mathcal{H}[E^x, E^\varphi; K_x, K_\varphi]$
- $\mathcal{H}_x[E^x, E^\varphi; K_x, K_\varphi]$

The framework – Symmetry reduction

Hamiltonian theory in Ashtekar's variables

- $\mathcal{H}[E_i^a; A_a^i]$
- $\mathcal{H}_\mathrm{i}[E_i^a; A_a^i]$
- $\mathcal{G}_\mathrm{i}[E_i^a; A_a^i]$

Spherical symmetry →

- $\mathcal{H}[E^x, E^\varphi; K_x, K_\varphi]$
- $\mathcal{H}_x[E^x, E^\varphi; K_x, K_\varphi]$

$$\mathcal{H} = \mathcal{H}^{(g)} + \mathcal{H}^{(m)}$$
$$\mathcal{H}_x = \mathcal{H}_x^{(g)} + \mathcal{H}_x^{(m)}$$

The framework – Gauge fixing

$$ds^2 = -N^2 dt^2 + \left(\frac{E^\varphi}{E^x}\right)^2 (dr + N^r\, dt)^2 + E^x d\Omega^2$$

The framework – Gauge fixing

$$ds^2 = -N^2 dt^2 + \left(\frac{E^\varphi}{E^x}\right)^2 (dr + N^r\, dt)^2 + E^x d\Omega^2$$

Areal gauge: $\chi = E^x - r^2$ (second class with $\mathcal{H}_x$)

The framework – Gauge fixing

$$ds^2 = -N^2 dt^2 + \left(\frac{E^\varphi}{E^x}\right)^2 (dr + N^r\, dt)^2 + E^x d\Omega^2$$

Areal gauge: $\chi = E^x - r^2$ (second class with $\mathcal{H}_x$)

$\mathcal{H}[E^\varphi; K_\varphi]$ → $\dot{A} = \left\{ A\,, \int dx\, \mathcal{H} \right\}$

The framework – Gauge fixing

$$ds^2 = -N^2 dt^2 + \left(\frac{E^\varphi}{E^x}\right)^2 (dr + N^r\, dt)^2 + E^x d\Omega^2$$

Areal gauge: $\chi = E^x - r^2$ (second class with $\mathcal{H}_x$)

$$\mathcal{H}[E^\varphi; K_\varphi \to \frac{1}{\mu}\sin \mu K_\varphi] \longrightarrow \dot{A} = \left\{A\,, \int dx\, \mathcal{H}\right\}$$

[Kelly, Santacruz, Wilson-Ewing (2020)]
[Kelly, Santacruz, Wilson-Ewing (2022)]
[Husain, Kelly, Santacruz, Wilson-Ewing (2022)]

Semiclassical Lemaitre-Tolman-Bondi model

$$\dot{r}^2 = N^2\left(\frac{2Gm}{r} + \varepsilon\right)\left[1 - \frac{\gamma^2\Delta}{r^2}\left(\frac{2Gm}{r} + \varepsilon\right)\right]$$

$$\dot{m} = -\,4\pi\Pi\, r^2\dot{r}$$

$$\dot{\varepsilon} = \frac{(1+\varepsilon)}{r'}\frac{2\dot{r}}{\rho+\Pi}\left[\frac{2r'}{r}(\Sigma - \Pi) - \Pi'\right]$$

$$\frac{N'}{N} = \frac{1}{\rho+\Pi}\left[\frac{2r'}{r}(\Sigma - \Pi) - \Pi'\right]$$

$$\dot{\Pi} = \frac{\delta\Pi}{\delta\rho}\dot{\rho}$$

$$\dot{\Sigma} = \frac{\delta\Sigma}{\delta\rho}\dot{\rho}$$

[Wilson-Ewing (2024)]

[Cafaro, Cipriani, Fazzini, Soltani (2025)]

Dust – The Oppenheimer-Snyder model

Dust: fluid with no pressure, $T^{\mu}{}_{\nu} = \rho u^{\mu} u_{\nu}$

OS: "homogeneous ball in a vacuum", $\rho(\mathrm{t}, \mathrm{R}) = \rho(\mathrm{t})\, \theta(\mathrm{R}_0 - \mathrm{R})$

Dust – The Oppenheimer-Snyder model

Dust: fluid with no pressure, $T^{\mu}{}_{\nu} = \rho u^{\mu} u_{\nu}$

OS: "homogeneous ball in a vacuum", $\rho(\mathrm{t}, \mathrm{R}) = \rho(\mathrm{t})\, \theta(\mathrm{R}_0 - \mathrm{R})$

$N' = 0$ → $a^{\mu} = u^{\nu} \nabla_{\nu} u^{\mu} = \frac{1 + \varepsilon}{(r')^2} \frac{N'}{N} \delta^{\mu}_{R} = 0$ → Dust follows geodesics

$\dot{m} = 0$ → No fluxes inside the "star"

$\dot{\varepsilon} = 0$

Dust – The Oppenheimer-Snyder model

$$\chi_k(R) = \frac{1}{\sqrt{k}} sin(\sqrt{k}R)$$

$$r(t, R) = a(t)\chi_k(R)$$

Dust – The Oppenheimer-Snyder model

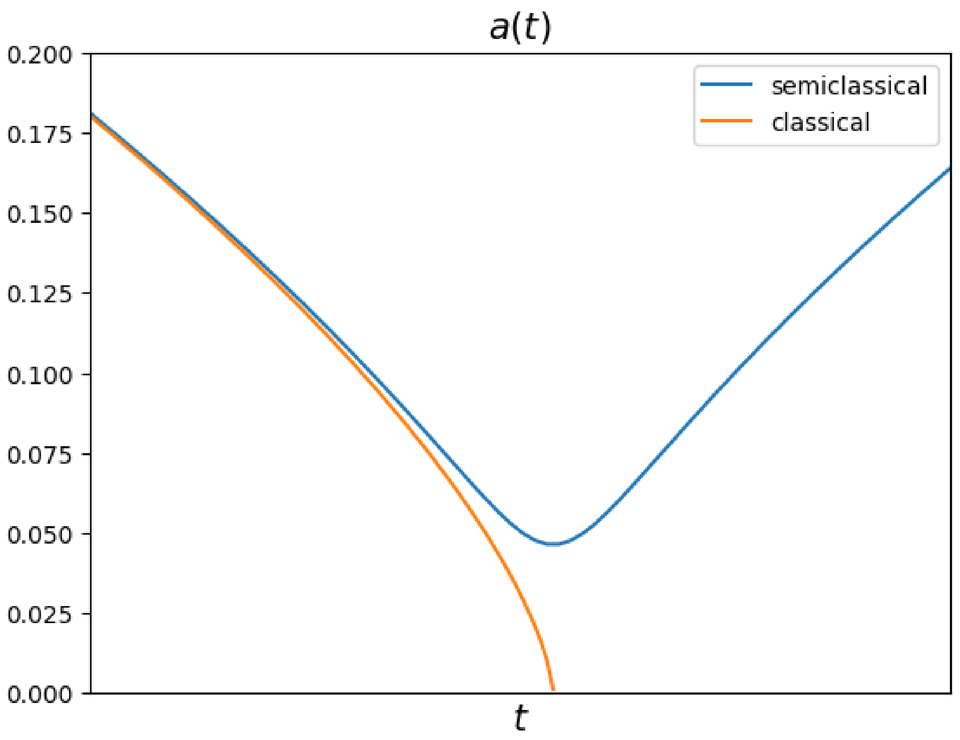
$$\chi_k(R) = \frac{1}{\sqrt{k}} sin(\sqrt{k}R)$$

$$r(t,R) = a(t)\chi_k(R)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\rho - \frac{k}{a^2}\right)\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi G\rho_c}\frac{k}{a^2}\right)$$

$$\rho = \frac{\rho_0}{a^3}$$

k = 0



Dust – The Oppenheimer-Snyder model

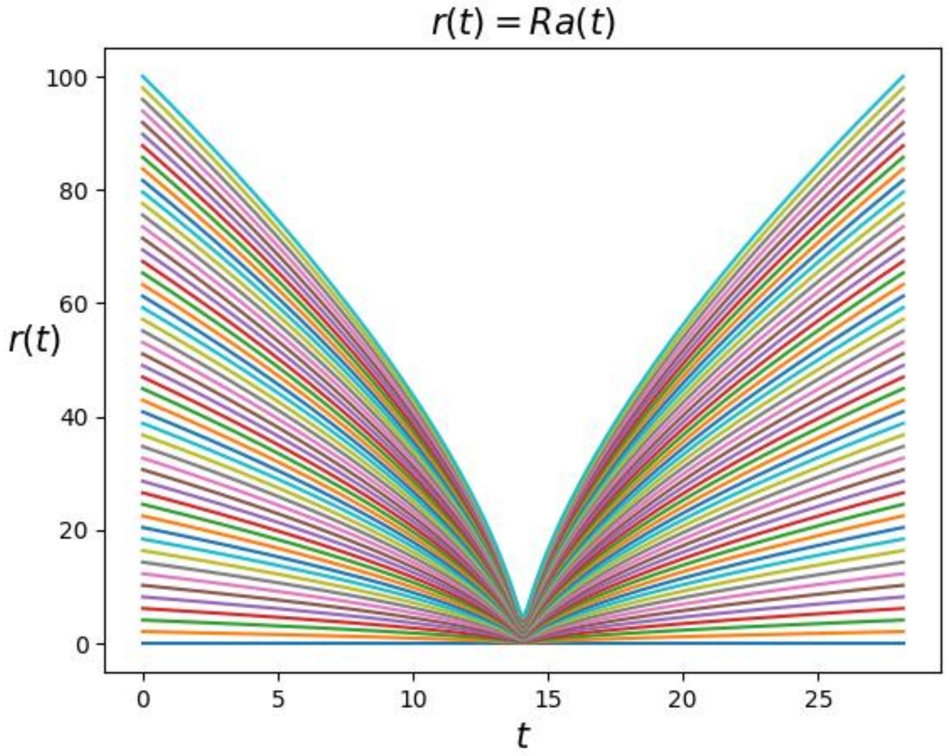
$$\chi_k(R) = \frac{1}{\sqrt{k}} sin(\sqrt{k}R)$$

$$r(t, R) = a(t)\chi_k(R)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\rho - \frac{k}{a^2}\right)\left(1 - \frac{\rho}{\rho_c} + \frac{3}{8\pi G\rho_c}\frac{k}{a^2}\right)$$

$$\rho = \frac{\rho_0}{a^3}$$

k = 0



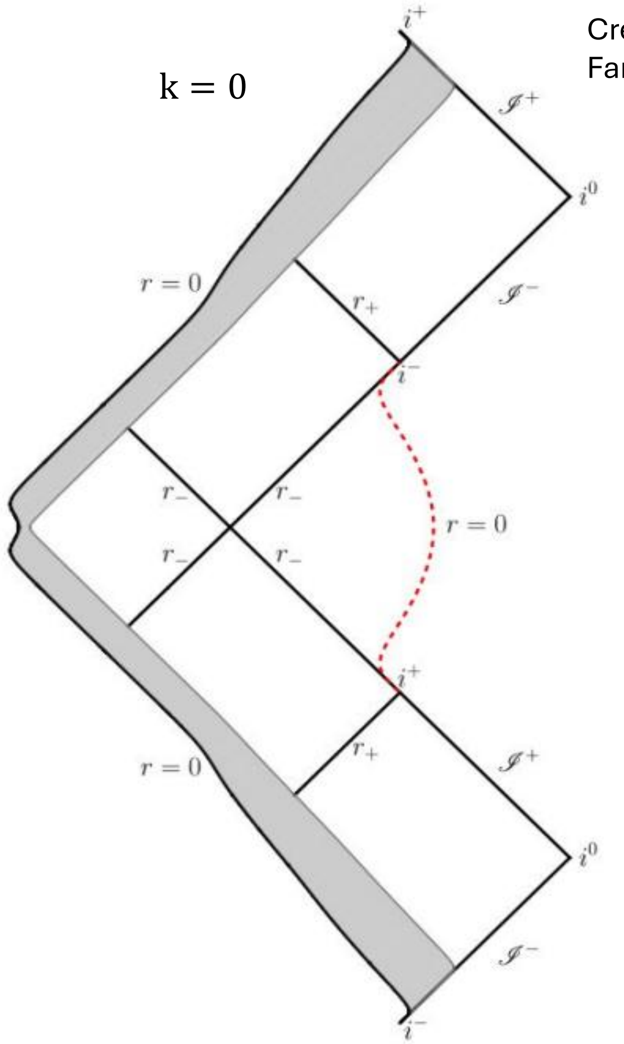
Dust – The Oppenheimer-Snyder model

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2GM}{r} + \frac{4\gamma^2\Delta}{r^2}\left(\frac{GM}{\mathrm{r}} - \frac{\mathrm{k}\,\chi_{\mathrm{k},0}^2}{2}\right)^2$$

[Kelly, Santacruz, Wilson-Ewing (2020)]
[Bobula, Pawlowski (2023)]
[Lewandowski, Ma, Yang, Zhang (2023)]
[Fazzini, Rovelli, Soltani (2023)]
[Cafaro, Lewandowski (2024)]

k = 0



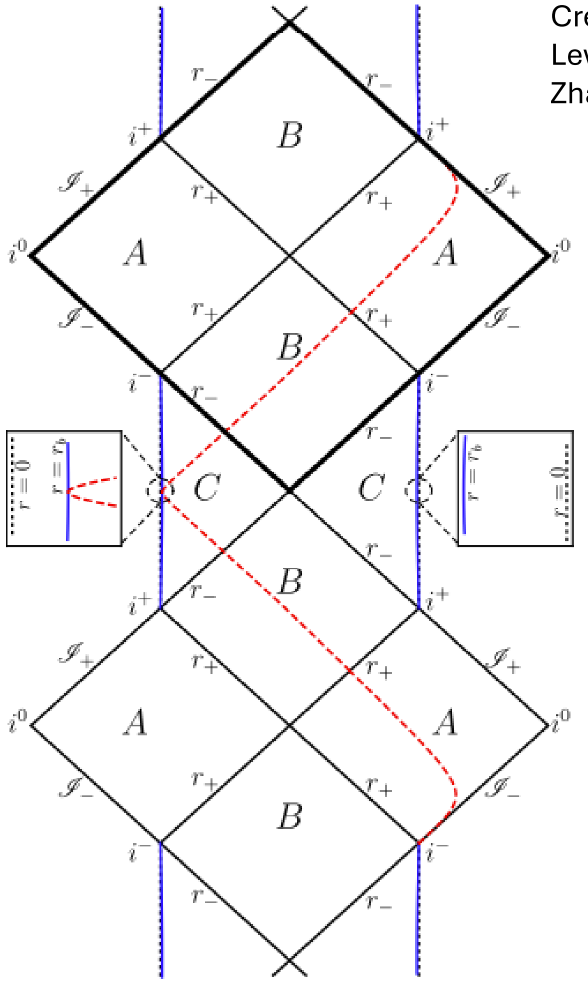
Credits:
Farshid Soltani

Dust – The Oppenheimer-Snyder model

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2GM}{r} + \frac{4\gamma^2\Delta}{r^2}\left(\frac{GM}{\mathrm{r}} - \frac{\mathrm{k}\,\chi_{\mathrm{k},0}^2}{2}\right)^2$$

[Kelly, Santacruz, Wilson-Ewing (2020)]
[Bobula, Pawlowski (2023)]
[Lewandowski, Ma, Yang, Zhang (2023)]
[Fazzini, Rovelli, Soltani (2023)]
[Cafaro, Lewandowski (2024)]

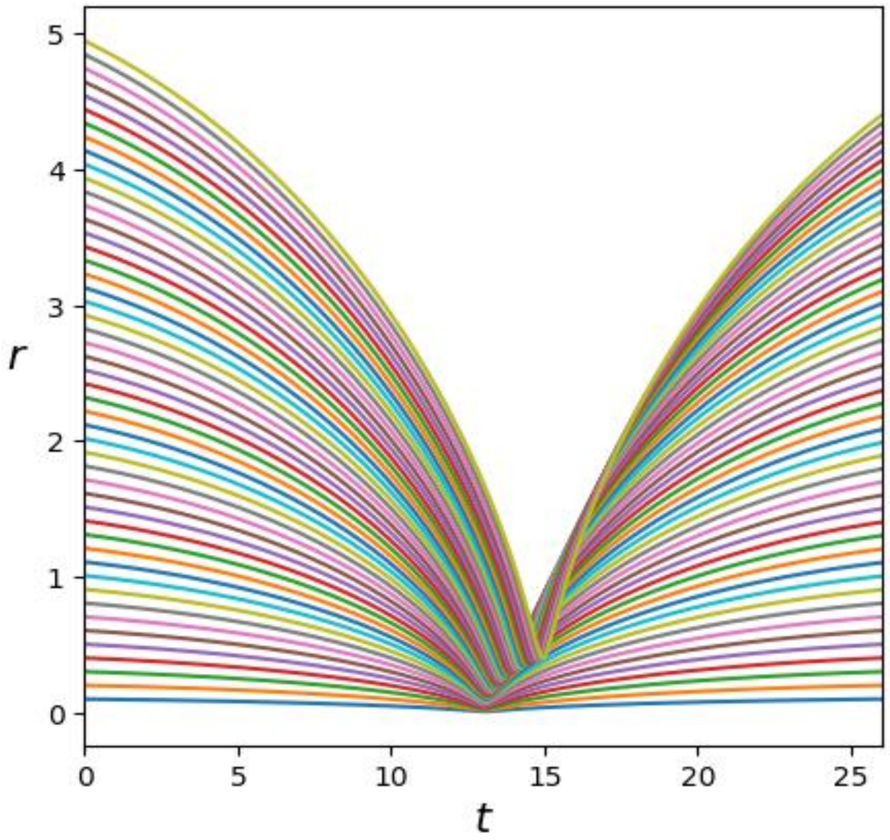


Credits:
Lewandowski, Ma, Yang, Zhang

Dust – Beyond the OS model

Non-homogeneous dust: $\rho(0,\mathrm{R}) = \rho_0(\mathrm{R})$

$$\left(\frac{\dot{r}}{r}\right)^2 = \left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\left[1 - \gamma^2 \Delta \left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\right]$$



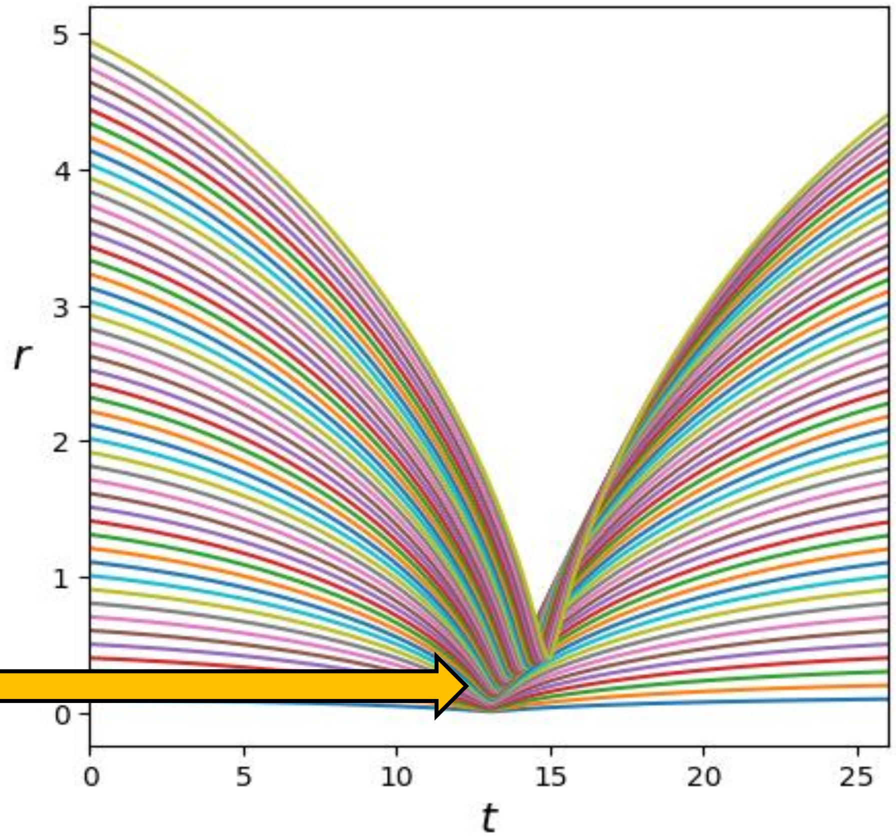
Dust – Beyond the OS model

Non-homogeneous dust: $\rho(0, \mathrm{R}) = \rho_0(\mathrm{R})$

$$\left(\frac{\dot{r}}{r}\right)^2 = \left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\left[1 - \gamma^2 \Delta \left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\right]$$

Shell Crossing singularity

$$\rho = \frac{\partial_R m}{4\pi r^2 \partial_R r} \to \infty$$



Dust – Beyond the OS model

[Nolan (2003)]
[Fazzini, Husain, Wilson-Ewing (2023)]
[Cipriani, Fazzini, Wilson-Ewing (2024)]

$$\partial_t u + \partial_r f(u, r) = 0$$

$$\frac{dL}{dt} = \frac{[f(u,r)]}{[u]}$$ with $[A] = \lim_{r \to L_+} A - \lim_{r \to L_-} A$

Rankine-Hugoniot condition

Dust – Beyond the OS model

[Nolan (2003)]
[Fazzini, Husain, Wilson-Ewing (2023)]
[Cipriani, Fazzini, Wilson-Ewing (2024)]

$$\partial_t u + \partial_r f(u,r) = 0$$

$$\frac{dL}{dt} = \frac{[f(u,r)]}{[u]} \quad \text{with } [A] = \lim_{r\to L_+} A - \lim_{r\to L_-} A$$

Rankine-Hugoniot condition

Weak solutions:

- Huge ambiguity
- Shock moves spacelike in some regions

Dust – Beyond the OS model

[Nolan (2003)]
[Fazzini, Husain, Wilson-Ewing (2023)]
[Cipriani, Fazzini, Wilson-Ewing (2024)]

$$\partial_t u + \partial_r f(u,r) = 0$$

$$\frac{dL}{dt} = \frac{[f(u,r)]}{[u]} \quad \text{with } [A] = \lim_{r \to L_+} A - \lim_{r \to L_-} A$$

Rankine-Hugoniot condition

Weak solutions:

- Huge ambiguity
- Shock moves spacelike in some regions

Israel junction condition at $\mathrm{r} = \mathrm{L(t)}$

$$\mathrm{ds}_+^2 = -\mathrm{g}_{\mathrm{tt}}^+ \mathrm{dt}_+^2 + \mathrm{L}_+^2 \mathrm{d}\Omega^2$$

$$\mathrm{ds}_-^2 = -\mathrm{g}_{\mathrm{tt}}^- \mathrm{dt}_-^2 + \mathrm{L}_-^2 \mathrm{d}\Omega^2$$

PG time is discontinuous

[Fazzini, Mehmood (2025)]

Perfect fluid

Perfect fluid: $T^{\mu}{}_{\nu} = \rho u^{\mu} u_{\nu} + \mathrm{P}\left(\delta^{\mu}_{\nu} + u^{\mu} u_{\nu}\right)$

EOS: $P(t,R) = \omega\rho(t,R)$

$$\dot{r}^2 = N^2\left(\frac{2Gm}{r} + \varepsilon\right)\left[1 - \frac{\gamma^2\Delta}{r^2}\left(\frac{2Gm}{r} + \varepsilon\right)\right]$$

$$\dot{m} = -4\pi p\, r^2 \dot{r}$$

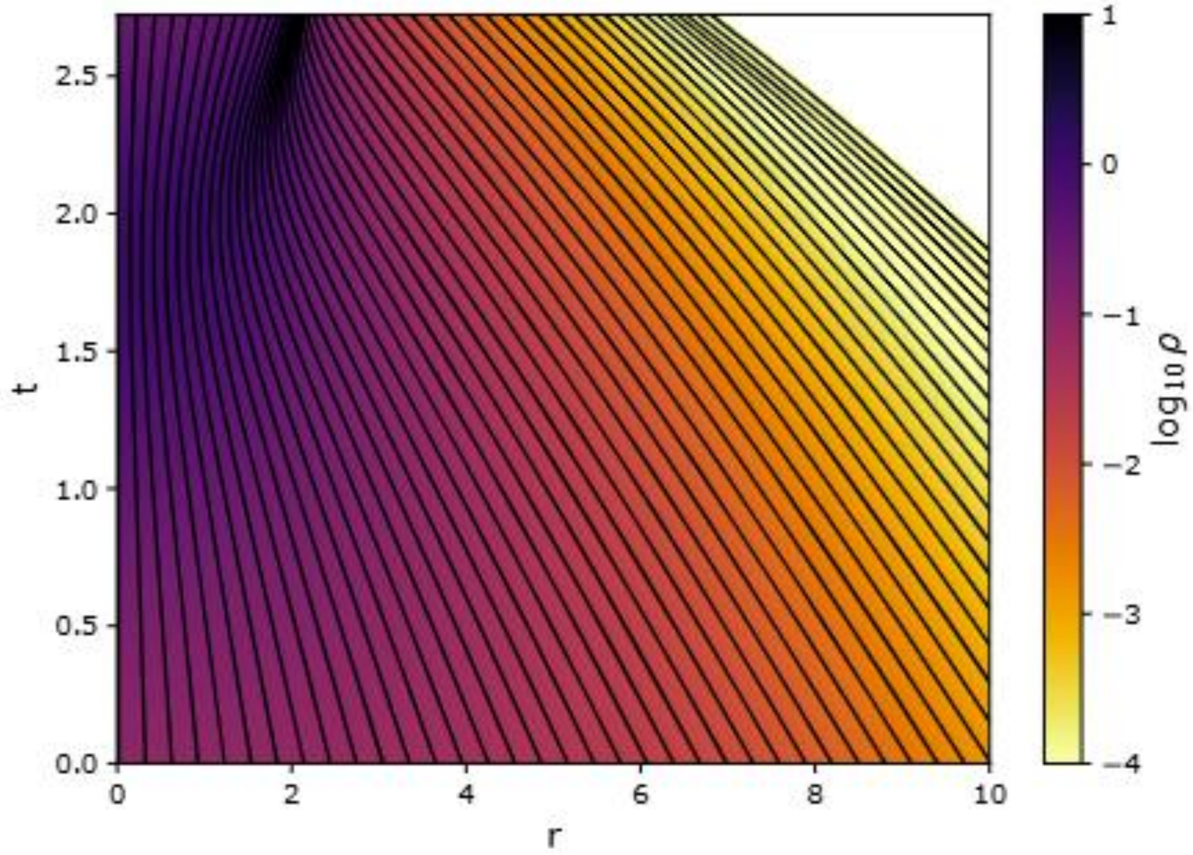
$$\dot{\varepsilon} = -2(1+\varepsilon)\frac{p'}{\rho + p}\frac{\dot{r}}{r'}$$

$$\frac{N'}{N} = -\frac{p'}{\rho + p}$$

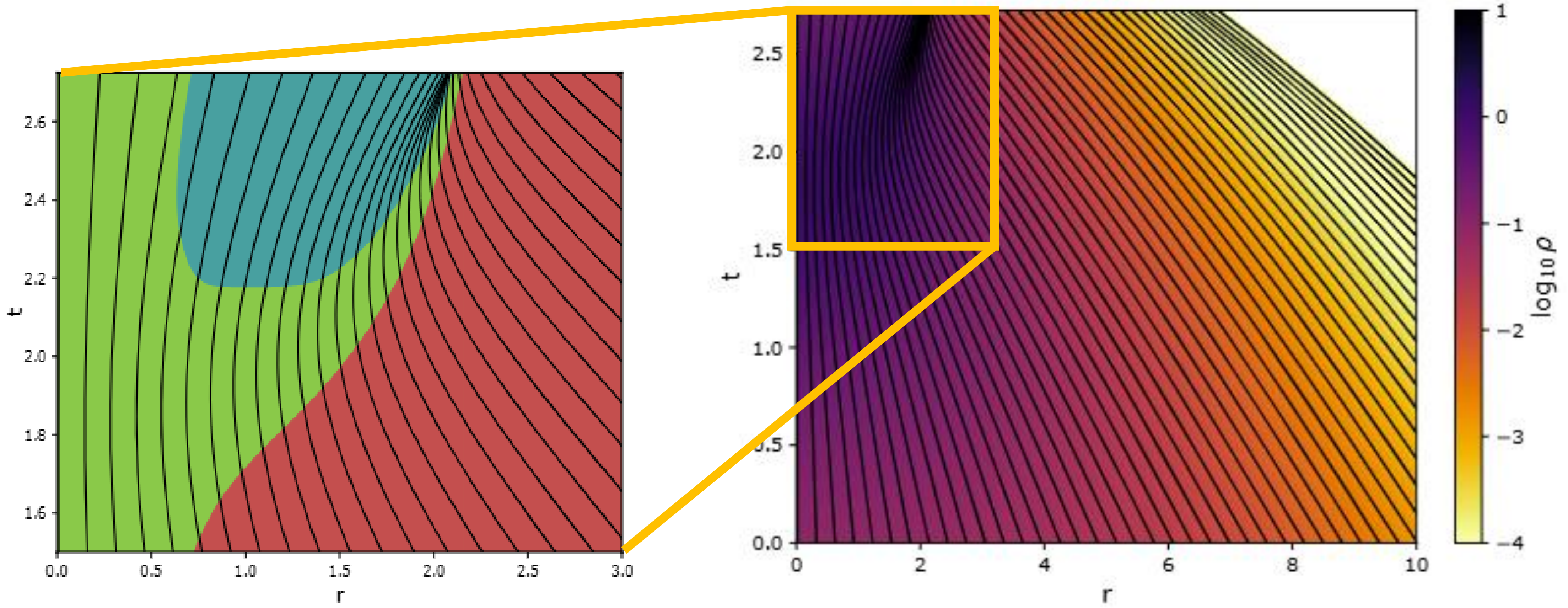
$$p = \omega\rho$$

[Cafaro, Cipriani, Fazzini, Soltani (2025)]

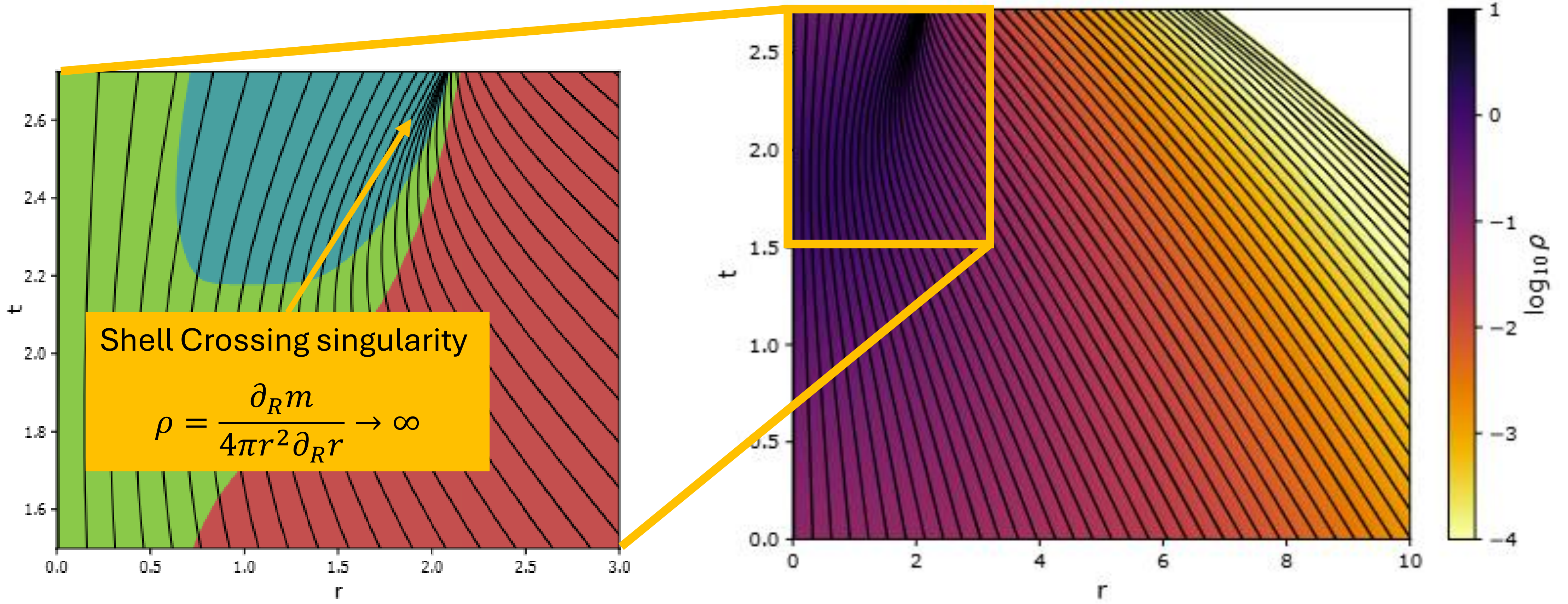
Perfect fluid



Perfect fluid



Perfect fluid



Perfect fluid – Vacuum

In Schwarzschild coordinates:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2GM}{r} + \frac{\gamma^2 \Delta}{r^2}\left(\frac{2GM}{r} + \varepsilon(r)\right)^2$$

Perfect fluid – Vacuum

In Schwarzschild coordinates:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2GM}{r} + \frac{\gamma^2 \Delta}{r^2}\left(\frac{2GM}{r} + \varepsilon(r)\right)^2$$

In the dust case, this is a constant

Conclusions

- Singularity resolution:

  Singularity replaced by a bounce (similar to LQC)

- SCS are still present:

  How do we remove the infinity (weak singularity)?

- Future of the shock? Hawking radiation? Angular momentum?